

Homework #12 due September 19

Read pp. 42 - 59.

Need to know: Definitions 2.2.3 and 2.2.3 B, 2.3.1. Statements and proofs of theorems 2.3.2, 2.3.3, 2.3.4, 2.4.2.

Do the following problems:

1. Prove that the sequence $\{\sin(\frac{n\pi}{2})\}_{n=1}^{\infty}$ does not have a limit.

2. Suppose that $\lim_{n \rightarrow \infty} b_n = b$ and $b \neq 0$.

a) Prove that there exists an $N \in \mathbb{N}$ such that $\forall n \in \mathbb{N}$, $|b_n| > |b|/2$.

b) Prove that $\lim_{n \rightarrow \infty} \frac{1}{b_n} = \frac{1}{b}$.

c) How do we know now that if $\lim_{n \rightarrow \infty} a_n = a$, then $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{a}{b}$?

3. Let $a_1 = 2$ and $a_{n+1} = \sqrt{2 + a_n}$, $n = 1, 2, \dots$

a) Find a_2, a_3, a_4 .

b) Prove by induction that $a_{n+1} \geq a_n$ and that $a_n \leq 2$ for all $n \in \mathbb{N}$.

c) Find $\lim_{n \rightarrow \infty} a_n$ (Hint: explain why the limit exists. Then take the limit on both sides of $a_{n+1} = \sqrt{2 + a_n}$.)

4. Do # 2.3.7.