List of Real Analysis topics for Final Exam

Final Exam is cumulative.

Topics from Tests 1, 2, and 3 to review

1. Definitions of least upper bound, and greatest lower bound of a set. Axiom of completeness. Characterization of the least upper bound (Lemma 1.3.8).

2. Theorem 1.4.1 (Nested Interval Property) with proof.

3. Theorem 1.4.2 (Archimedian Property) with proof.

4. Definitions 2.2.3 and 2.2.3 B of convergence of a sequence.

5. Definition of convergence of a series, sum of an infinite geometric progression.

6. Bolzano-Weierstrass Theorem (with proof).

7. Definition of Cauchy sequence. Sequence converges if and only if it is Cauchy (Cauchy Criterion) (with proof).

8. Cauchy Criterion for series. Necessary condition of convergence (Theorem 2.7.3). Comparison test (Theorem 2.7.4). Absolute convergence test. Alternating series test. Ratio test. Definitions of absolute and conditional convergence.

9. Definitions of an open set, limit point, isolated point, closed set, closure of a set, complement of a set. Theorem 3.2.3 (unions and intersections of open sets, with proof), Theorem 3.2.5 (characterization of a limit point), Theorem 3.2.13, Theorem 3.2.14.

10. Definitions of a compact set, bounded set, open cover, finite subcover. Theorem 3.3.4 (Characterization of compactness, with proof), Theorem 3.3.5 (nested compact sets), Theorem 3.3.8 (Heine – Borel theorem).

11. Definitions 4.4.1 and 4.4.1B (limit of a function). Theorem 4.2.3 (Sequential criterion for functional limits), Corollary 4.2.5 (Divergence criterion for functional limits)

12. Definition 4.3.1 (continuity at a point), Theorem 4.3.2 (Characterization of Continuity), Corollary 4.3.3 (Criterion for discontinuity), Theorem 4.3.4 (Algebraic continuity theorem), Theorem 4.3.9 (Composition of continuous functions, with proof).

13. Definition 4.4.4 (uniform continuity). Theorem 4.4.1 (Preservation of compact Sets, with proof), Theorem 4.4.2 (Extreme value theorem, with proof), Theorem 4.4.5 (Sequential criterion for nonuniform continuity), Theorem 4.4.7 (Uniform continuity on a compact set, with proof).

14. Intermediate value theorem.

15. Definition of derivative. Theorem 5.2.3 (Differentiability implies continuity, with proof), Theorem 5.2.4 (Properties of differentiability, with proof), Theorem 5.2.5 (Chain rule, with proof), Theorem 5.2.6 (Interior Extremum Theorem), Theorem 5.2.7 (Darboux's theorem)

16. Theorem 5.3.1 (Rolle's Theorem, with proof), Theorem 5.3.2 (Mean value theorem, with proof).

17. Definitions of pointwise and uniform convergence of a family of functions. Theorem 6.2.5 (Cauchy Criterion for Uniform Convergence), Theorem 6.2.6 (Uniform limit of continuous functions is continuous, with proof).

18. Theorem 6.3.3 (with proof from class).

19. Definition 6.4.1 (pointwise and uniform convergence of a series of functions). Theorems 6.4.2, 6.4.3, 6.4.4. Corollary 6.4.5 (Weierstrass M-test with proof).

20. Possibly some material from 6.5.