Homework # 4 (Due Wednesday, February 5)

- There will be a quiz on February 5. For the quiz you need to know statements and proofs of propositions 3.3, 3.4, 3.5; theorem 6.1.
- Read Chapters 3 and 6 in the textbook.

Do the following problems:

1. Consider the equation $ax^2+bx+c=0$, where a, b, and c are integers. (An example of such an equation is $3x^2+4x+1=0$, but we are not specifying a, b, and c). Assume that $ax^2+bx+c=0$ has two real solutions x_1 and x_2.

Prove or find a counterexample:

(a) x_1 and x_2 are always rational numbers.

(b) If x_1 is an integer, then x_2 also must be an integer.

(c) If x_1 is a rational number, then x_2 also must be a rational number.

2. Do problem # 2 on p. 49.

(Hint: let $u = a+bi$ and $v = c+di$, where a, b, c, d are real numbers. For each equality in #2 express and compare both sides in terms of a, b, c, and d).

3. a) Prove that for all $z, w \in \mathbb{C}$,

$$|z+w|^2 = |z|^2 + |w|^2 + 2\,\mathrm{Re}(z\bar{w}).$$

b) Prove that for all $z \in \mathbb{C}$,

$$|\mathrm{Re}\, z| \le |z| \text{ and } |\mathrm{Im}\, z| \le |z|.$$

(So, in particular, $\mathrm{Re}(z\bar{w}) \le |z\bar{w}|$)

c) Use a) and b) to prove that

for all $z, w \in \mathbb{C}$, $|z+w| \le |z| + |w|$.

4) Use trigonometric identities to show that $[\cos(\theta_1) + i\sin(\theta_1)] \cdot [\cos(\theta_2) + i\sin(\theta_2)]$

$= \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)$.

5) Write each complex number below in the form $z = r(\cos\theta + i\sin\theta)$, $0 \le \theta < 2\pi$.

a) $(\sqrt{3} - i)^{10}$; b) $(-2\sqrt{3} - 2i)^6$.