Homework # 24 due Monday 04/27.

You can rework problems from homework # 22 for full credit (also due Monday, 04/27).

1. Prove that for every $\varepsilon > 0$, there is a collection of intervals $\{I_n\}_{n=1}^{\infty}$, where each $I_n = [a_n, b_n]$, $b_n > a_n$, such that $\mathbb{Q} \subseteq \bigcup_{n=1}^{\infty} I_n$ and the sum of lengths of all the intervals is less than ε (i.e. $\sum_{n=1}^{\infty} \ell(I_n) \leq \varepsilon$) Write a complete proof.

(Hint: Since $\mathbb{Q}$ is countable, we can arrange all rational numbers as a list $\mathbb{Q} = \{q_1, q_2, \ldots, q_n, \ldots\}$ Now cover each q_n by an interval I_n of length $\frac{1}{2^{n+1}}$ centered at q_n. What are the endpoints of I_n? What $\sum_{n=1}^{\infty} \ell(I_n)$ equals to ?)

2. (a) Let S be the set consisting of all the finite subsets of $\mathbb{N}$. Prove that S is countable.

(b) Let T be the set consisting of all infinite subsets of $\mathbb{N}$. Prove that T is uncountable.

(c) Explain why your solution of problem 4 in homework 23, shows that the power set of $\mathbb{N}$ is uncountable.