# Homework # 23 (due Friday, 04/24)

1. Let $A_1, A_2, \ldots, A_n$ be countable sets.

a) Assume that the sets $A_1, \ldots, A_n$ are disjoint, i.e. $A_i \cap A_j = \emptyset$. Use only the description of a countable set as an infinite list to show that $B = A_1 \cup A_2 \cup \ldots \cup A_n$ is also countable.

b) Now do not assume that sets $A_1, A_2, \ldots, A_n$ are disjoint. Describe how you would arrange the set $B = A_1 \cup A_2 \cup \ldots \cup A_n$ as an infinite list.

2. By definition, $card(A) \leq card(B)$ if and only if there is a one-to-one function $g: A \to B$. Also, $card(A) \geq card(B)$ if there is an onto function $h: A \to B$. Prove that

a) $card(A) \leq card(B)$ and $card(B) \leq card(C) \Rightarrow card(A) \leq card(C)$;

b) $card(A) \geq card(B)$ and $card(B) \geq card(C) \Rightarrow card(A) \geq card(C)$.

3. Prove that the set $I$ of all irrational numbers is uncountable. (Hint. the shortest proof is by contradiction.)

4. Prove that the set of all infinite sequences of zeros and ones is uncountable. (Two examples of such sequences are 101000... and 1010010001.....)
Hint. Modify the Cantor diagonalization argument.