Homework # 22 (due Wednesday, April 22)

1. Let S, T be two sets and $f: S \to T$ is a bijection. Recall that then f has an inverse function $g: T \to S$, defined as follows: $\forall t \in T$, $g(t) = s_0$ where s_0 is the unique solution of the equation $f(s) = t$.
   Prove that $g. T \to S$ is a bijection.

2. Let $A_1, A_2, \dots, A_n, \dots$ be countable sets. Prove that $\bigcup_{n=1}^{\infty} A_n$ is countable. (Hint. For each set A_n, since A_n is countable we can arrange A_n as a list

$$A_n = \{a_{n1}, a_{n2}, \dots, a_{nm}, \dots \}.$$

Now describe a way of arranging countably many lists into a single list, the proof is similar to the proof that $\mathbb{Q}_+$ is countable) <u>Write a complete proof.</u>