Homework #21 (Due Monday, April 20)

1. Let $\sim$ be a relation on $\mathbb{Z}$, defined by $m \sim n$ if $m-n$ is divisible by 5.

a) Verify that the relation above is an equivalence relation (i.e. check 3 properties in the definition of equivalence relation)

b) How many equivalence classes this relation has. Describe them.

2. Let S be any set and let $\sim$ be an equivalence relation on S. Suppose that $s_1 \sim s_2$ for some $s_1, s_2 \in S$. Use the definitions of equivalence relation and equivalence class to show that $cl(s_1) = cl(s_2)$. (Hint: Both $cl(s_1)$ and $cl(s_2)$ are subsets of S, what do you need to do to show that two sets are equal?)

~~3. Let $[a,b]$ and $[c,d]$ be two arbitrary intervals in $\mathbb{R}$. Find a bijection $f: [a,b] \to [c,d]$.~~

3. a) Suppose A and B are countable sets such that $A \cap B = \emptyset$. Prove that $A \cup B$ is also countable. (Set A is called countable if there is a bijection $f: \mathbb{N} \to A$)

b) Prove by induction that if $A_1, A_2, \ldots, A_n$ are countable sets with $A_i \cap A_j = \emptyset \ \forall\, i,j$, then $A_1 \cup A_2 \cup \ldots \cup A_n$ is also countable.