**Lecture:**

# Homework # 17 (due April 8)

1. (De Morgan's Laws). Let $A$ and $B$ be subsets of real numbers, $A_1, A_2, \ldots, A_n, \ldots$ also be subsets of $\mathbb{R}$.

(a) Prove that $(A \cap B)^c = A^c \cup B^c$.

(b) Use induction to prove that $(A_1 \cup A_2 \cup \ldots \cup A_n)^c = A_1^c \cap A_2^c \cap \ldots \cap A_n^c$ for all $n \in \mathbb{N}$.

(c) Prove that $\left(\bigcup_{i=1}^{\infty} A_i\right)^c = \bigcap_{i=1}^{\infty} A_i^c$ (Note: do not use induction, prove directly.

2. Given a function $f$ and a subset $A$ of its domain, let $f(A)$ represent the range of $f$ over the set $A$; that is $f(A) = \{f(x) \mid x \in A\}$.

(a) Let $f(x) = x^2$. $\mathbb{R} \to \mathbb{R}$. If $A = [0,2]$ and $B = [1,4]$, find $f(A)$ and $f(B)$. Does $f(A \cap B) = f(A) \cap f(B)$? Does $f(A \cup B) = f(A) \cup f(B)$?

(b) Find two sets $C$ and $D$ for which $f((C \cap D) \neq f(C) \cap f(D)$

(c) Show that for an arbitrary function $g: \mathbb{R} \to \mathbb{R}$, it is always true that $g(A \cap B) \leq g(A) \cap g(B)$ for all sets $A, B \subseteq \mathbb{R}$

(d) Is it always true that $g(A \cup B) = g(A) \cup g(B)$? If true, prove this. If false, find a counterexample.