Homework 15 due Wednesday, April 1.

1. Guess $\bigcup_{n=1}^{\infty} A_n$ and $\bigcap_{n=1}^{\infty} A_n$, where A_n is defined as follows for all $n \in \mathbb{N}$.

(a) $A_n = \{ x \in \mathbb{R} \mid x \leq n \}$;

(b) $A_n = \{ x \in \mathbb{R} \mid -\frac{1}{n} \leq x \leq \frac{1}{n} \}$;

(c) $A_n = \{ x \in \mathbb{R} \mid -\frac{1}{n} < x < n \}$.

2. Decide which of the following represent true statements about the nature of sets. For any that are false, provide a specific example where the statement in question does not hold. Prove the true statements.

(a) $A \cap (B \cup C) = (A \cap B) \cup C$;

(b) $A \cap (B \cap C) = (A \cap B) \cap C$;

(c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$;

(d) If $A_1 \supseteq A_2 \supseteq \ldots \supseteq A_n \supseteq \ldots$ are all sets containing an infinite number of elements, then the intersection $\bigcap\limits_{n=1}^{\infty} A_n$ is infinite as well.