Homework #11 (Due ~~Wednesday~~ Monday, March 16)

Please do the following problems:

1. Generalize the <u>second</u> proof of the Cauchy inequality from $n=3$ to an arbitrary n. I.e. prove that $|a_1b_1+\dots+a_nb_n| \le \sqrt{a_1^2+\dots+a_n^2}\cdot\sqrt{b_1^2+\dots+b_n^2}$ for all $n=1,2,\dots$. Don't forget to investigate the case of equality.

2. Prove the following inequalities. For each inequality, indicate when the inequality becomes an equality.

   a) $\tan\theta+\cot\theta \geqslant 2$, if $0<\theta<\pi/2$;

   b) $\sqrt{\dfrac{a+b}{a+b+c}}+\sqrt{\dfrac{a+c}{a+b+c}}+\sqrt{\dfrac{b+c}{a+b+c}} \le \sqrt{6}$, if $a>0, b>0, c>0$.

   c) $(a+b)(b+c)(c+a) \geqslant 8abc$, if $a>0, b>0, c>0$.

3. Prove that for all $a>0, b>0$ one has

$$\frac{2ab}{a+b}=\frac{2}{\frac{1}{a}+\frac{1}{b}} \le \sqrt{ab} \le \frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}} \le \max\{a,b\}.$$

Notice that you need to prove 4 inequalities.

4. Prove the following inequalities. Indicate when each inequality becomes an equality.

   a) $\dfrac{z}{y}+\dfrac{8x}{z}+\dfrac{27y}{x} \geqslant 18$, if $x>0, y>0, z>0$.

   b) $a^4+b^4+c^4+16d^4 \geqslant 8abcd$.