

Homework # 10 (due Wednesday, March 4)

Do the following problems:

1 a) Guess a formula for the product

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right)\dots\left(1 - \frac{1}{n^2}\right).$$

b) Use math induction to prove your guess.

2. Suppose $a + \frac{1}{a}$ is an integer. Use math

induction to prove that $a^n + \frac{1}{a^n}$ is also an integer for all $n = 1, 2, \dots$.

3. Use the method of math induction to prove that

a) $n! > 2^n$ for all $n \geq 4$;

b) $n! > n^2$ for all $n \geq 4$;

c) $\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \geq \frac{7}{12}$, $n \geq 2$.

4. Do # 7 on p. 74 (Hint: numbers

$\alpha = \frac{1+\sqrt{5}}{2}$ and $\beta = \frac{1-\sqrt{5}}{2}$ are the roots of

$x^2 - x - 1 = 0$ (check that!), so $\alpha^2 = \alpha + 1$ and $\beta^2 = \beta + 1$)