

3. In the previous homework you showed that $\mathcal{F}[e^{-|x|}] = \sqrt{\frac{2}{\pi}} \frac{1}{1+\lambda^2}$.

Verify the Plancherel Formula in this case, i.e. show that if $f(x) = e^{-|x|}$, then $\|f(x)\|^2 = \|\hat{f}(\lambda)\|^2$, where $\|\cdot\|$ is the $L^2(\mathbb{R})$ norm.

4. Let $h(t) = \begin{cases} e^{-2t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$ and $f(t) = \begin{cases} 1, & 0 \leq t \leq \pi, \\ 0, & \text{otherwise.} \end{cases}$

If H is the filter defined by

a) $H[f] = h * f$, find $H[f]$.

b) Find the system function $\sqrt{2\pi} \hat{h}(\lambda)$.