Homework due November 29.

(1) Let L be a linear, time invariant transformation such that

$$L[e^{i\lambda t}] = i\lambda e^{i\lambda t} \quad \forall \lambda \in \mathbb{R}$$

Let $f(t) = t^2$, $-\pi \le t \le \pi$, extended periodically with period 2π.

a) Find $L[f(t)]$.

<u>Hint</u>: Represent f as its complex Fourier series and apply L to both sides.

b) Show that $L[f(t)] = f'(t)$ when $-\pi < x < \pi$

<u>Hint</u>: Represent $f'(t)$ as a Fourier series.

(2) Let $f_\delta(t)$ be a δ-sequence of smooth functions as defined in class. Let $h(t)$ be an arbitrary continuous function, then

$$\lim_{\delta \to 0} \int_{-\infty}^{+\infty} f_\delta(t) h(t)\, dt = h(0).$$

<u>Hint</u>: Since $h(t)$ is continuous at $t = 0$, $\forall \varepsilon > 0 \ \exists \delta > 0$ s.t. $|h(t) - h(0)| < \varepsilon$ for all t, $|t| < \delta$.

Now show that $\lim_{\delta \to 0} \int_{-\infty}^{+\infty} f_\delta(t)(h(t) - h(0))\, dt = 0$.

(3) Use (2) to verify that

$$\lim_{\delta \to 0} \mathcal{F}[f_\delta(t-a)] = \frac{e^{-i\lambda a}}{\sqrt{2\pi}}$$