Hwk due November 8.

1. Show that the function $\hat{f}(\lambda) = \frac{1-\cos(\pi\lambda)}{\lambda^2}$ is infinitely differentiable

(Hint: Replace $\cos(\pi\lambda)$ with its Taylor series at $\lambda = 0$).

2. Let $h(x) = x e^{-x^2/2}$.

a) Find $\hat{h}(\lambda)$ (Hint: Use property 4 on page 102).

b) Show that $\hat{h}(x) = C \cdot h(x)$.

Find the constant C.

3. a) Calculate the Fourier transforms of $x^2 e^{-x^2/2}$ and $x^3 e^{-x^2/2}$.

b) Find constants α and β such that if $\phi(x) = (x^2+\alpha) e^{-x^2/2}$ and $\psi(x) = (x^3+\beta x) e^{-x^2/2}$, then $\hat{\phi}(x) = -\phi(x)$ and $\hat{\psi}(x) = i\psi$.

4. Prove that if f is an even function, then $\hat{f}$ is also even; and if f is odd, then $\hat{f}$ is also odd.

5. Let $F(\lambda) = \hat{f}(\lambda)$. Prove that $\hat{F}(\lambda) = f(-\lambda)$ (Hint: use Fourier inversion Formula)