Handout for section 1.2 (solutions)

Not on homework

#1 (b) Let $p(x) = x^4 - 2x^3 + 3x - 6$.
Then $p(1) = 1 - 2 + 3 - 6 = -4$, so $x = 1$ is not a solution;
$p(2) = 16 - 16 + 6 - 6 = 0$, so $x = 2$ is a solution
$p(-2) = 16 + 16 - 6 - 6 \neq 0$, so $x = -2$ is not a solution.

#2 b) • $dy_1/dt = 0 = -y_1^2$, so $y_1(t) = 0$ is a solution.

• $dy_2/dt = -\frac{1}{t^2} = -\left(\frac{1}{t}\right)^2 = -y_2^2$, so

$y_2(t) = \frac{1}{t}$ is a solution.

• $dy_3/dt = -\frac{2}{t^2} \neq -y_3^2 = -\frac{4}{t^2}$, so $y_3(t) = \frac{2}{t}$ is

not a solution.

• $dy_4/dt = -\frac{1}{(t-2)^2} = -y_4^2$, so $y_4(t) = \frac{1}{t-2}$ is a solution

#3b) Pluging in: $y'' + y' - 2y = \lambda^2 e^{\lambda t} + \lambda e^{\lambda t} - 2e^{\lambda t}$
$= e^{\lambda t}(\lambda^2 + \lambda - 2) = 0 \iff \lambda^2 + \lambda - 2 = (\lambda + 2)(\lambda - 1) = 0$
$\boxed{\lambda_1 = -2, \ \lambda_2 = 1}$

#4c) Let $y(t) = c$ be a constant solution then
$dy/dt = 0 = (t^2 - 1)(c-5)\cos c$, so
$c = 5$ or $\cos c = 0 \Rightarrow c = 5$ or $\frac{\pi}{2} + \pi n$
$n = 0, \pm 1, \pm 2, \ldots$

#5(b) Let $y(t) = at + b$, then

$$\frac{dy}{dt} = a = 3(at+b) - t + 2, \text{ so}$$

$$a = t(3a-1) + 3b + 2 \text{ for all } t.$$

Thus $\begin{cases} 3a - 1 = 0 & \Rightarrow a = \frac{1}{3} \\ a = 3b+2 & \Rightarrow b = -5/9 \end{cases}$ $\boxed{y(t) = \frac{1}{3} t - 5/9}$

Initial condition: $y(0) = -5/9$.

#6(b) Many different answers are possible, for example,

- $dy/dt = 3t^2 + 1$
- $dy/dt = \dfrac{3y}{t} - 2$
- $dy/dt = \dfrac{y}{t} + 2t^2$