Homework (solutions)

#1 (a) x and y are competing species since interaction terms $-4xy$ and $-6xy$ are both negative.

(b) From 2^{nd} equation we have

$\frac{dy}{dt} = 8y(1 - y/2)$. This is a logistic equation with carrying capacity 2. Phase line is

. Every solution starting above $y = 2$ will decrease to 2, every solution starting between 0 and 2 will increase to 2.

(c) Equilibrium points satisfy

$$\begin{cases} 3x(1 - \frac{x}{3}) - 4xy = 0 \\ 8y(1 - y/2) - 6xy = 0 \end{cases} \Rightarrow \begin{cases} x(3 - x - 4y) = 0 \\ y(8 - 4y - 6x) = 0 \end{cases}$$

From the first equation: $x = 0$ or $x + 4y = 3$.

If $x = 0$, 2nd equation becomes $8y(1 - y/2) = 0$, so $\boxed{(x, y) = (0,0) \text{ or } (0,2).}$

If $x + 4y = 3$, then from the 2^{nd} equation $y = 0$ or $6x + 4y = 8$. Thus $\boxed{(x,y) = (3,0)}$ or

$\begin{cases} x + 4y = 3 \\ 6x + 4y = 8 \end{cases}$ and $\boxed{(x,y) = (1, \frac{1}{2})}$.

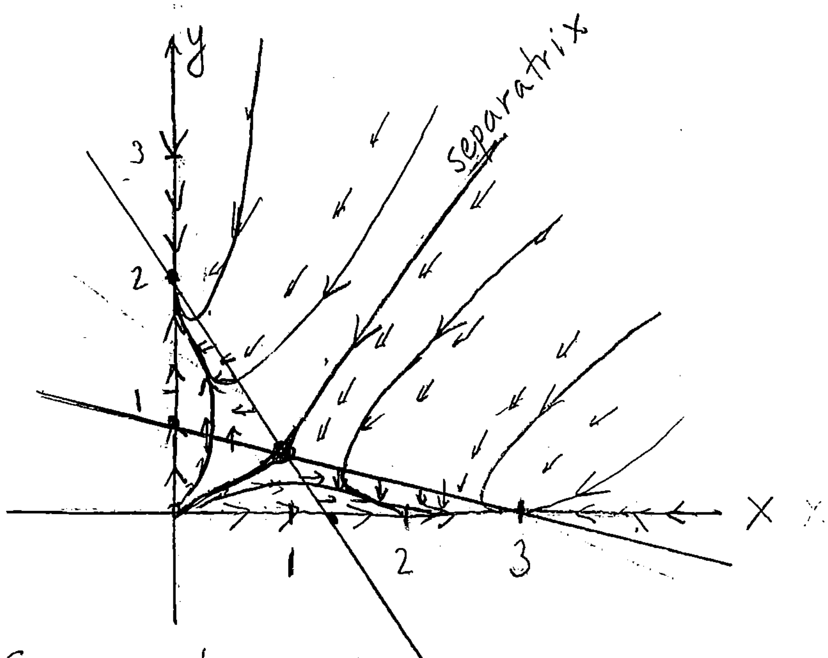
(d) Vector field $F(x,y) = \left(3x(1-\frac{x}{3}) - .4xy,\ 8y(1-\frac{y}{2}) - 6xy\right)$.

F is vertical when $3x(1-\frac{x}{3}) - 4xy = x(3 - x - 4y) = 0$.

Vertical nullclines are $\boxed{x = 0 \text{ or } x + 4y = 3}$.

Horizontal nullclines satisfy $8y(1-y/2) - 6xy =$

$= y(8 - 4y - 6x) = 0$; $\boxed{y = 0 \text{ or } 4y + 6x = 8}$



(e) See picture above

(f) $x_1(t)$ will decrease to zero, $y_1(t)$ will approach 2;
$x_2(t)$ will also decrease to zero, $y_2(t)$ will increase to 2.

(g) Every solution curve starting on separatrix will approach equilibrium point $(1, \frac{1}{2})$.
Every solution curve that starts below the separatrix will approach equilibrium point $(3, 0)$.
Every solution curve that starts above separatrix will approach equlibrium point $(0, 2)$.

#2 (a)

$$y'' + 5y' + 4y = k^2 e^{kt} + 5k e^{kt} + 4e^{kt} =$$
$$= e^{kt}(k^2 + 5k + 4) = 0 \Rightarrow k^2 + 5k + 4 = (k+1)(k+4) = 0$$

$k_1 = -1$, $k_2 = -4$.

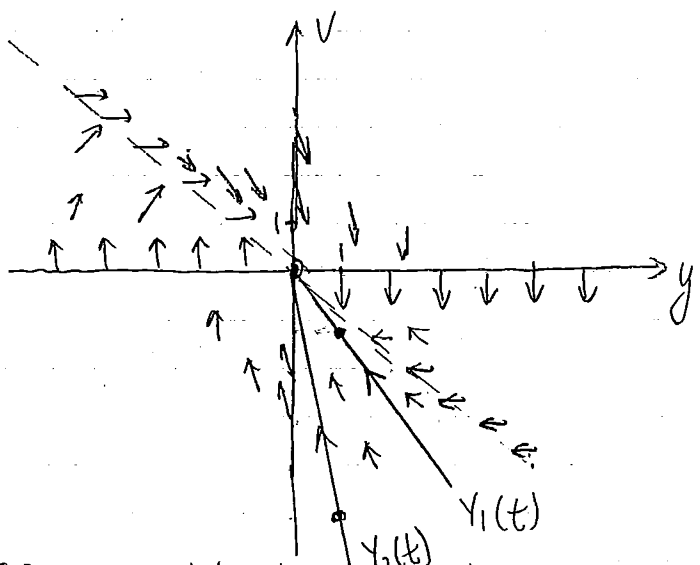
Two solutions are $y_1(t) = e^{-t}$ and $y_2(t) = e^{-4t}$.

(b) $$\begin{cases} dy/dt = v \\ \dfrac{dv}{dt} = -4y - 5v \end{cases}$$

(c) Vector field is $F(y, v) = (v, -4y - 5v)$

Horizontal nullcline is $-4y - 5v = 0$ or $v = -\frac{4}{5}y$

Vertical nullcline is $v = 0$



(d) Straight-line solutions are

$$Y_1(t) = \begin{pmatrix} y_1(t) \\ v_1(t) \end{pmatrix} = \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix} \text{ and } Y_2(t) = \begin{pmatrix} y_2(t) \\ v_2(t) \end{pmatrix} = \begin{pmatrix} e^{-4t} \\ -4e^{-4t} \end{pmatrix}$$

$$Y_1(t) = e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad Y_2(t) = e^{-4t}\begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

e) See picture above.