Homework due Tuesday; October 18.

1. Consider the system of two differential equations representing two interacting species

$$\begin{cases} \frac{dx}{dt} = 3x(1 - x/3) - 4xy \\ dy/dt = 8y(1 - y/2) - 6xy \end{cases}$$

a) Describe the type of interaction. Possible answers are: predator-prey, cooperating species, or competing species. Please explain.

b) Suppose $x(t) = 0$ for all t. What will happen to y in the long run? Your answer should include the picture of phase line for y.

c) Find all equilibrium points. Show all your work.

d) Find horizonal field and vertical field nullclines for the system. Draw the nullclines on the (x,y)-plane. On each nullcline draw vectors of the direction field.

e) Use HPG system solver program to draw direction field in the window $-1 \le x \le 4$, $-1 \le y \le 4$. Draw phase portrait in that window.

f) Use your phase portrait to decide what happens to solutions starting at $x_1(0) = 2$, $y_1(0) = 3$? $x_2(0) = \frac{1}{2}$, $y_2(0) = 1$?

g) Describe all possible fates of solutions starting at $x(0) = a$, $y(0) = b$, where a and b are positive numbers.

2. Consider the differential equation

$$y'' + 5y' + 4y = 0$$

a) Find two different solutions of this equation in the form $y(t) = e^{kt}$.

b) Convert the differential equation into a system of two equations in variables y and v.

c) Draw the direction field for the system in b).

d) Find two straight-line solutions of the system (you should get them straight from a))

e) Draw solution curves of the two solutions from d) on the phase plane.