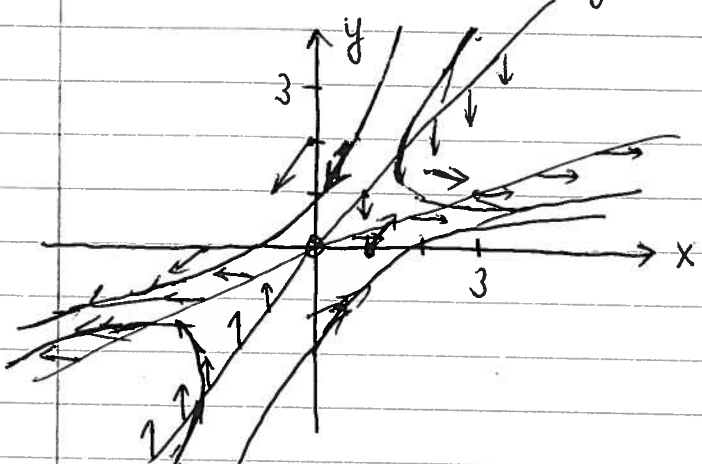
Solutions to hwk due 10/12/2018

\# 1 a) Vector field $\vec{F}(x,y) = (x-y\ ,\ x-3y)$.

Horizontal nullcline: $x-3y=0$ or $y = x/3$

Vertical nullcline: $x-y=0$ $\quad y = x$.

$\vec{F}(1,0) = (1,1)$

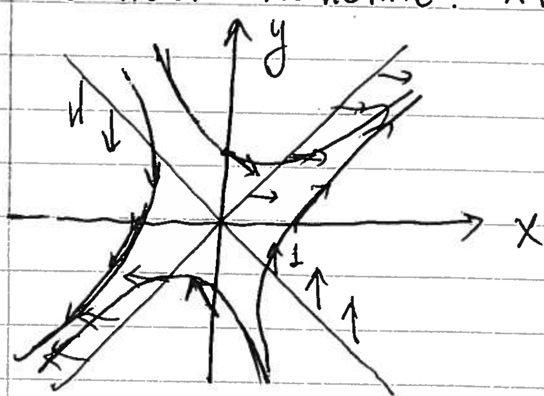


b) Vector field $\vec{F}(x,y) = (x+y\ ,\ x-y)$

Horizontal nullcline: $x-y=0$ or $y=x$

Vertical nullcline: $x+y=0$ or $y=-x$

$F(1,0) = (1,1)$



2

2. a) Plug in:

$(e^{st})'' + 4(e^{st})' + 3e^{st} = s^2 e^{st} + 4s e^{st} + 3e^{st} = 0$

$e^{st}(s^2 + 4s + 3) = 0 \Rightarrow s^2 + 4s + 3 = 0$

$s^2 + 4s + 3 = (s+1)(s+3) = 0 \Rightarrow s_1 = -1, \; s_2 = -3$

Two different solutions are $y_1(t) = e^{-t}$, $y_2(t) = e^{-3t}$.

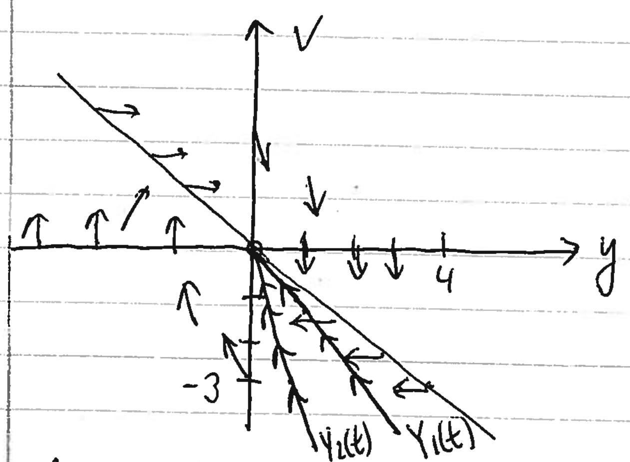
b) $\begin{cases} dy/dt = v \\ dv/dt = -3y - 4v \end{cases}$

$\frac{dv}{dt} = y'' = -4y' - 3y$ from diff. equation

c) Vector field $\vec{F}(y, v) = (v, -3y - 4v)$

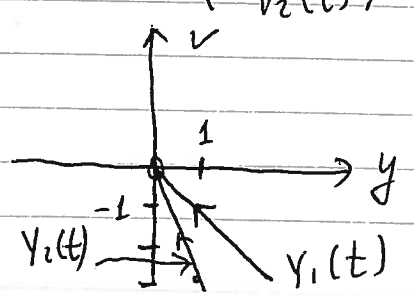
Horizontal nullcline: $-3y - 4v = 0$, $v = -\frac{3}{4}y$.

Vertical nullcline: $v = 0$



d) $\vec{Y}_1(t) = \begin{pmatrix} y_1(t) \\ v_1(t) \end{pmatrix} = \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix} = e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

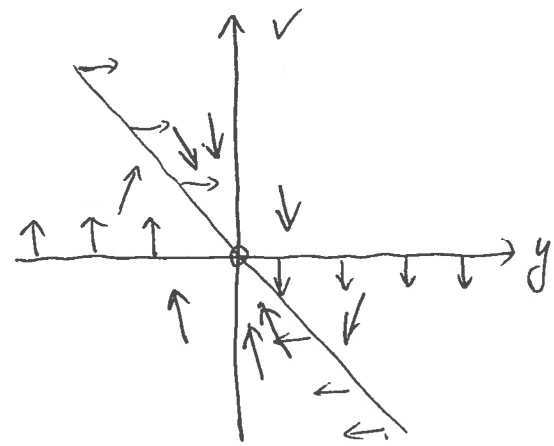
$\vec{Y}_2(t) = \begin{pmatrix} y_2(t) \\ v_2(t) \end{pmatrix} = \begin{pmatrix} e^{-3t} \\ -3e^{-3t} \end{pmatrix} = e^{-3t}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$.



Solution to problems from Sec. 2.3

#2 (a) Associated system

$$\begin{cases} dy/dt = v \\ dv/dt = -6y - 5v \end{cases}$$



Horiz. nullcline $-6y-5v=0$ or

$v = -\frac{6}{5}y$

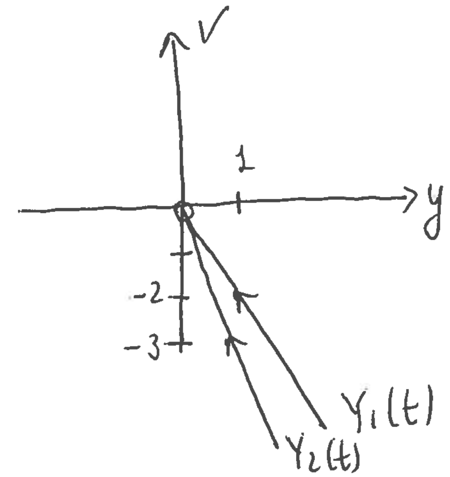
b) Plug in $y = e^{st}$:

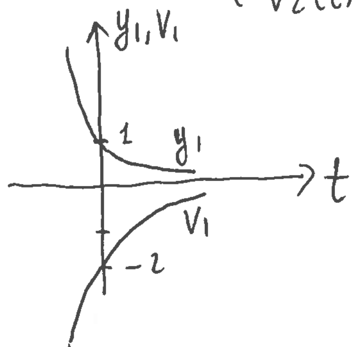
$e^{st}(s^2+5s+6)=0 \Rightarrow s_1=-2,\ s_2=-3$

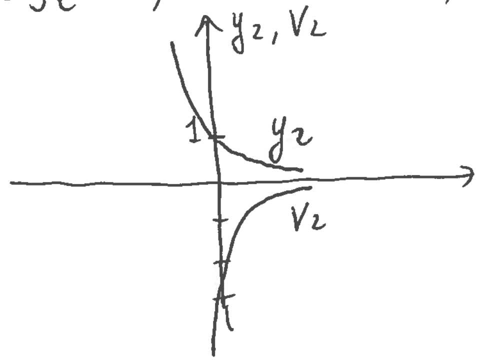
$y_1(t) = e^{-2t} \quad y_2(t) = e^{-3t}$

c) $Y_1(t) = \begin{pmatrix} y_1(t) \\ v_1(t) \end{pmatrix} = \begin{pmatrix} e^{-2t} \\ -2e^{-2t} \end{pmatrix} = e^{-2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$Y_2(t) = \begin{pmatrix} y_2(t) \\ v_2(t) \end{pmatrix} = \begin{pmatrix} e^{-3t} \\ -3e^{-3t} \end{pmatrix} = e^{-3t}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

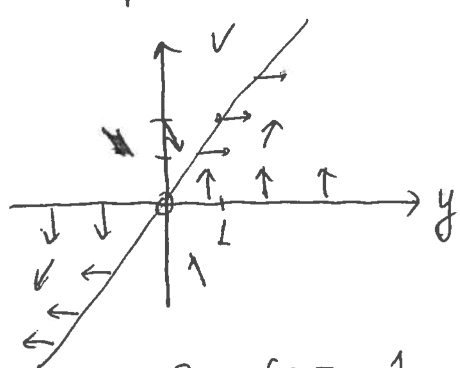






#6 Associated system:

$$\begin{cases} dy/dt = v \\ dv/dt = 2y - v \end{cases}$$

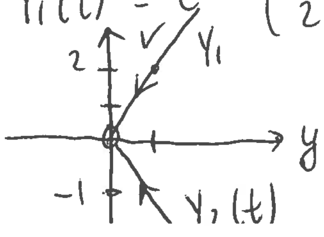


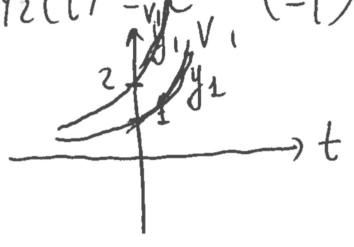
Horiz. nullcline: $2y - v = 0$

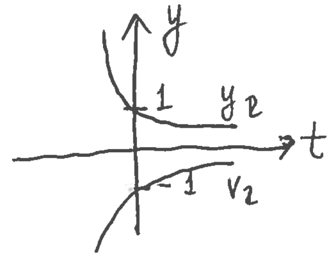
Vert. nullcline: $v = 0$

$s^2+s-2 = (s-2)(s+1)=0 \quad s_1 = 2,\ s_2 = -1,\ y_1(t) = e^{2t},\ y_2(t) = e^{-t}$

$Y_1(t) = e^{2t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad Y_2(t) = e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$







There is a solution $y_1(t)$ that goes to $+\infty$ as $t \to \infty$

Not a harmonic oscillator

#8 p. 187. (a) Let $y_1(t)$ and $y_2(t)$ be two solutions of $my'' + by' + ky = 0$. We check that $z(t) = y_1(t) + y_2(t)$ is also a solution of the same differential equation.

$$mz'' + bz' + kz = m(y_1+y_2)'' + b(y_1+y_2)' + k(y_1+y_2)$$

$$= my_1'' + my_2'' + by_1' + by_2' + ky_1 + ky_2 =$$

$$= [my_1'' + by_1' + ky_1] + [my_2'' + by_2' + ky_2] = 0 + 0 = 0$$

(Expressions in square brackets are both 0 since y_1 and y_2 are solutions). Since $z(t)$ satisfies DE, it is a solution.

b) $y'' + 3y' + 2y = 0$ has two solutions $y_1(t) = e^{-2t}$ and $y_2(t) = e^{-t}$. By a) $y(t) = y_1(t) + y_2(t) = e^{-2t} + e^{-t}$ is a solution too. $y(0) = 2$, $y'(0) = -3$ obeys initial conditions.

c) One can check that $y(t) = 2e^{-2t} + e^{-t}$ also is a solution (Plug it in!). It obeys $y(0) = 3$, $y'(0) = -5$.

d) If C_1 and C_2 are arbitrary constans,

$$C_1 y_1(t) + C_2 y_2(t) = C_1 e^{-2t} + C_2 e^{-t}$$ is a solution of $y'' + 3y' + 2y = 0$ $\Rightarrow$ This DE has infinitely many solutions.