Homework # 7 (Solutions)

Sec. 1.1.

21. (a) The term governing the effect of the interaction of x and y on the rate of change of x is $+\beta xy$. Since this term is positive, the presence of y's helps the x population grow. Hence, x is the predator. Similarly, the term $-\delta xy$ in the dy/dt equation implies that when $x > 0$, y's grow more slowly, so y is the prey. If $y = 0$, then $dx/dt < 0$, so the predators will die out; thus, they must have insufficient alternative food sources. The prey has no limits on its growth other than the predator since, if $x = 0$, then $dy/dt > 0$ and the population increases exponentially.

(b) Since $-\beta xy$ is negative and $+\delta xy$ is positive, x suffers due to its interaction with y and y benefits from its interaction with x. Hence, x is the prey and y is the predator. The predator has other sources of food than the prey since $dy/dt > 0$ even if $x = 0$. Also, the prey has a limit on its growth due to the $-\alpha x^2/N$ term.

22. (a) We consider dx/dt in each system. Setting $y = 0$ yields $dx/dt = 5x$ in system (i) and $dx/dt = x$ in system (ii). If the number x of prey is equal for both systems, dx/dt is larger in system (i). Therefore, the prey in system (i) reproduce faster if there are no predators.

(b) We must see what effect the predators (represented by the y-terms) have on dx/dt in each system. Since the magnitude of the coefficient of the xy-term is larger in system (ii) than in system (i), y has a greater effect on dx/dt in system (ii). Hence the predators have a greater effect on the rate of change of the prey in system (ii).

(c) We must see what effect the prey (represented by the x-terms) have on dy/dt in each system. Since x and y are both nonnegative, it follows that

$$-2y + \tfrac{1}{2}xy < -2y + 6xy,$$

Sec 2.1

1. In the case where it takes many predators to eat one prey, the constant in the negative effect term of predators on the prey is small. Therefore, (ii) corresponds the system of large prey and small predators. On the other hand, one predator eats many prey for the system of large predators and small prey, and, therefore, the coëfficient of negative effect term on predator-prey interaction on the prey is large. Hence, (i) corresponds to the system of small prey and large predators.

2. For (i), the equilibrium points are $x = y = 0$ and $x = 10$, $y = 0$. For the latter equilibrium point prey alone exist; there are no predators. For (ii), the equilibrium points are $(0, 0)$, $(0, 15)$, and $(3/5, 30)$. For the latter equilibrium point, both species coexist. For $(0, 15)$, the prey are extinct but the predators survive.

Finding equilibrium points:

(i) $\begin{cases} 10x\left(1-\frac{x}{10}\right) - 20xy = x(10 - x - 20y) = 0 \\ -5y + \frac{xy}{20} = y\left(-5 + \frac{x}{20}\right) = 0 \end{cases}$

$\begin{cases} x = 0 \text{ or } 10 - x - 20y = 0 \\ y = 0 \text{ or } -5 + \frac{x}{20} = 0 \end{cases}$

$\begin{cases} x=0 \\ y=0 \end{cases}$ $\begin{cases} 10 - x - 20y = 0 \\ y = 0 \end{cases}$ $\begin{cases} 10 - x - 20y = 0 \\ -5 + x/20 = 0 \end{cases}$

$(0,0)$ $(10, 0)$ $(100, -9/2)$

This equil. point does not represent biological situation

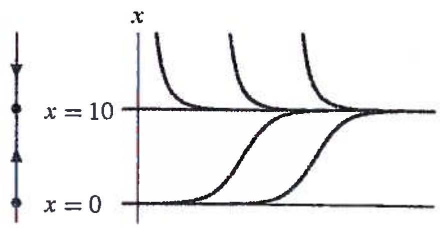
(ii) $\begin{cases} 0.3x - \frac{xy}{100} = x\left(0.3 - \frac{y}{100}\right) = 0 \\ 15y\left(1-\frac{y}{15}\right) + 25xy = y(15 - y + 25x) \end{cases}$

$\begin{cases} x = 0 \text{ or } 0.3 - \frac{y}{100} = 0 \\ y = 0 \text{ or } 15 - y + 25x = 0 \end{cases}$

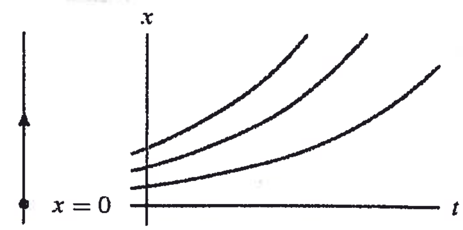
$\begin{cases} x=0 \\ y=0 \end{cases}$ $\begin{cases} x = 0 \\ 15 - y + 25x = 0 \end{cases}$ $\begin{cases} 0.3 - y/100 = 0 \\ 15 - y + 25x = 0 \end{cases}$

$(0,0)$ $(0, 15)$ $(3/5, 30)$.

4. For (i), the prey obey a logistic model. The population tends to the equilibrium point at $x = 10$. For (ii), the prey obey an exponential growth model, so the population grows unchecked.

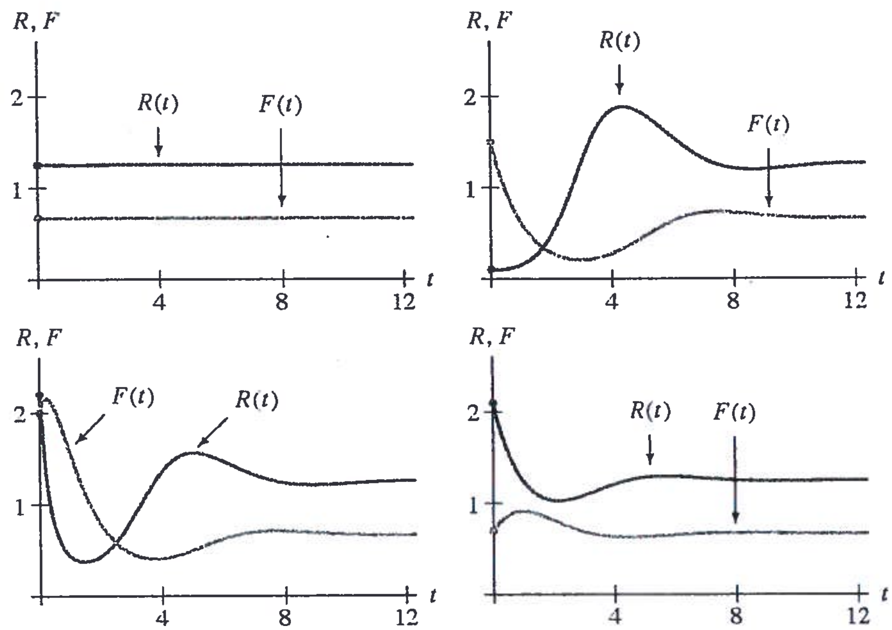


Phase line and graph for (i).



Phase line and graph for (ii).

8. **(a)**



(b) Each of the solutions tends to the equilibrium point at $(R, F) = (5/4, 2/3)$. The populations of both species tend to a limit and the species coexist. For curve B, note that the F-population initially decreases while R increases. Eventually F bottoms out and begins to rise. Then R peaks and begins to fall. Then both populations tend to the limit.

10. Hunting decreases the number of predators by an amount proportional to the number of predators alive (that is, by a term of the form $-kF$), so we have $dF/dt = -F + 0.9RF - kF$ in each case.

12. In the absence of prey, the predators would obey a logistic growth law. So we could modify both systems by adding a term of the form $-kF/N$, where k is the growth-rate parameter and N is the carrying capacity of predators. That is, we have $dF/dt = kF(1 - F/N) + 0.9RF$.

16. At first, the number of rabbits decreases while the number of foxes increases. Then the foxes have too little food, so their numbers begin to decrease. Eventually there are so few foxes that the rabbits begin to multiply. Finally, the foxes become extinct and the rabbit population tends to the constant population $R = 3$.