Solutions to problems from hwk 4.

1. Plug in $y(t) = e^{at} + bt^2$ into $\frac{dy}{dt} = 3y - 6t^2 + 4t$:

$$\frac{d}{dt}\left(e^{at} + bt^2\right) = 3\left(e^{at} + bt^2\right) - 6t^2 + 4t$$

$$a e^{at} + 2bt = 3e^{at} + 3bt^2 - 6t^2 + 4t \text{ for all } t.$$

Comparing right and left sides we see that $\boxed{a = 3 \text{ and } b = 2}$

2. a) Separating variables: $\int \frac{dy}{y^3} = \int t\,dt$,

$$-\frac{1}{2y^2} = \frac{t^2}{2} + C, \quad y^2 = -\frac{1}{t^2 + 2C}, \quad \boxed{\begin{cases} y = \pm\sqrt{-1/(t^2+2C)} \\ \text{OR} \\ y = 0 \text{ (equil. solution)} \end{cases}}$$

b) • If $y(0) = 1$, we want $y(0) = \pm\sqrt{-1/2C} = 1$.
We choose "+" and $C = -\frac{1}{2}$. $\Rightarrow \boxed{y(t) = \sqrt{\frac{-1}{t^2-1}}}$

• If $y(0) = -1$, $y(t) = \boxed{-\sqrt{-1/(t^2-1)}}$

• If $y(0) = 0$, then $\boxed{y(t) = 0}$ (equil. solution).

3. Let $y(t)$ be the amount of chocolate in gal. at a time t in min.

a) $\frac{dy}{dt} = 2 - 2 \cdot \frac{y}{53}$, $y(0) = 3$.

b) The volume is changing with time as $53 + 2t$. Thus

$$\frac{dy}{dt} = 4 - 2\frac{y}{53+2t}, \quad y(0) = 3.$$