Homework # 17

1. Verify the extended linearity principle, i.e. verify that if $y_p(t)$ is a solution of $y'' + py' + qy = f(t)$ and if $K_1 y_1(t) + K_2 y_2(t)$ is a solution of $y'' + py' + qy = 0$, then $y_p(t) + K_1 y_1(t) + K_2 y_2(t)$ satisfies $y'' + py' + qy = f(t)$.

2. For each differential equation below

a) Find the general solution.

b) Find a particular solution satisfying $y(0)=0$ and $y'(0)=0$.

c) Identify transient / steady-state parts of the solution in b), also identify free / forced response parts of the solution.

I. $y'' + 4y' + 5y = t + e^{-2t}$;

II. $y'' + 4y' + 3y = \cos(2t)$;

III $y'' + 9y = te^{-t}$.

3. Consider the differential equation $y'' + 4y' + 4y = e^{-2t}$

a) Verify that $y_1(t) = e^{-2t}$ and $y_2(t) = te^{-2t}$ are solutions of the equation $y'' + 4y' + 4y = 0$

b) Find a particular solution of $y'' + 4y' + 4y = e^{-2t}$ (Hint: try $y_p(t) = t^2 e^{-2t}$)