Homework 16

Sunday, November 11, 2018 8:19 PM

1. Suppose $y_1(t)$ and $y_2(t)$ are both solutions of a differential equation $y'' + py' + qy = 0$.

   Verify that $K_1 y_1(t) + K_2 y_2(t)$ is also a solution of the same differential equation.

2. Suppose that $y(t) = y_{re}(t) + i\, y_{im}(t)$ is a complex solution of a differential equation $y'' + py' + qy = 0$. Verify that $y_{re}(t)$ and $y_{im}(t)$ are two solutions of the same equation. (Hint: plug in and write the result as (expression for y_{re}) $+ i$ (expression for y_{im}) $= 0$. Then use the fact that complex number is zero if and only if its real and imaginary parts are zero).

3. For each differential equation below

   a) Identify differential equation as underdamped, overdamped, undamped, or critically damped.

   b) Find the general solution in real form. If applicable, find the natural period and natural frequency of the solution.

   c) Find the solution satisfying $y(0) = 4$, $y'(0) = -2$.

   d) Convert differential equation to the corresponding system and draw the phase portrait of the system. Find the general solution of the system.

   e) On a separate graph draw the solution curve for the solution in c).

   f) Graph $y(t)$ from c). If applicable, compute the amplitude of the solution.

Differential equations:

I. $y'' + 2y' + 10y = 0$

II. $y'' + 6y' + 5y = 0$