Homework #14

1. a) It is known that matrix $\begin{pmatrix} 1 & b \\ c & 3 \end{pmatrix}$ has an eigenvalue $\lambda_1 = 1+\sqrt{3}$. Find the second eigenvalue λ_2.

b) Find the product $b \cdot c$.

2. Draw phase portraits for the systems below. Identify each portrait as a sink, source, or a saddle. Label eigenvector directions as fast and slow.

a) $\begin{cases} dx/dt = 3x + 2y, \\ dy/dt = -3x - 4y. \end{cases}$ b) $\begin{cases} dx/dt = 3x + y, \\ dy/dt = x + 3y. \end{cases}$

c) System, corresponding to $y'' + 3y' + 2y = 0$.

3. Let $\begin{cases} dx/dt = 4x + y, \\ dy/dt = -x + 4y. \end{cases}$

a) Find complex eigenvalues and eigenvectors.

b) Find the general solution in complex form.

c) If $x(0) = 1$, $y(0) = 2$, find the particular solution $Y(t)$ in complex form.

d) Verify that $\overline{Y(t)} = Y(t)$.

4. Write $z = \sqrt{3} - i$ as $z = re^{i\theta}$.