Homework # 13 handout

Problem 1. (Special matrices). For each matrix below, find its eigenvalues and eigenvectors.

(a) $A = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$;

(b) $A = \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix}$, $a \neq 0, c \neq 0$;

(c) $A = \begin{pmatrix} 1 & 2 \\ 0 & 5 \end{pmatrix}$;

(d) $A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$, $a \neq 0, b \neq 0, c \neq 0$;

e) A matrix $A = \begin{pmatrix} 0 & 1 \\ -5 & -6 \end{pmatrix}$ for the system $\begin{cases} dy/dt = v \\ dv/dt = -5y - 6v \end{cases}$ associated to $y'' + 6y' + 5y = 0$

Problem 2. Find the number a, so that the vector $V = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ is an eigenvector of the matrix $A = \begin{pmatrix} a & 1 \\ 2 & 3 \end{pmatrix}$. What is the eigenvalue for this vector?

Problem 3. Find the general solutions of the following systems:

a) $\begin{cases} dx/dt = 4x - 2y, \\ dy/dt = x + y. \end{cases}$ b) $\frac{dY}{dt} = AY$, $A = \begin{pmatrix} 2 & 2 \\ 2 & 1 \end{pmatrix}$.