Homework # 12 (Solutions)

1 (a) Plug in:

- $\frac{d}{dt}\left(e^{5t}\begin{pmatrix}1\\1\end{pmatrix}\right) \overset{?}{=} \begin{pmatrix}4 & 1\\2 & 3\end{pmatrix}\left(e^{5t}\begin{pmatrix}1\\1\end{pmatrix}\right)$

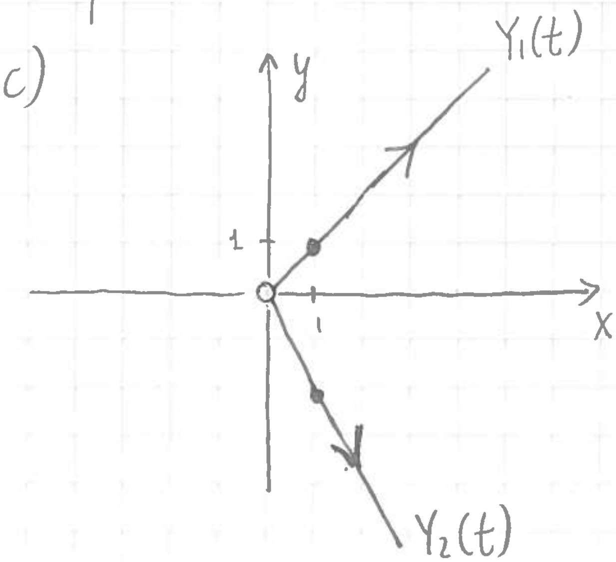
$$5e^{5t}\begin{pmatrix}1\\1\end{pmatrix} = e^{5t}\begin{pmatrix}4 & 1\\2 & 3\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = e^{5t}\begin{pmatrix}5\\5\end{pmatrix} \checkmark$$

- $\frac{d}{dt}\left(e^{2t}\begin{pmatrix}1\\-2\end{pmatrix}\right) \overset{?}{=} \begin{pmatrix}4 & 1\\2 & 3\end{pmatrix}\left(e^{2t}\begin{pmatrix}1\\-2\end{pmatrix}\right)$

$$2e^{2t}\begin{pmatrix}1\\-2\end{pmatrix} = e^{2t}\begin{pmatrix}4 & 1\\2 & 3\end{pmatrix}\begin{pmatrix}1\\-2\end{pmatrix} = e^{2t}\begin{pmatrix}4-2\\2-6\end{pmatrix} = e^{2t}\begin{pmatrix}2\\-4\end{pmatrix} \checkmark$$

b) $Y_1(0) = \begin{pmatrix}1\\1\end{pmatrix}$, $Y_2(0) = \begin{pmatrix}1\\-2\end{pmatrix}$ are linearly indep., that $Y_1(t)$ and $Y_2(t)$ are linearly indep. too.

c)



d) By the linearity principle the gen. solution is

$$Y(t) = c_1 e^{5t}\begin{pmatrix}1\\1\end{pmatrix} + c_2 e^{2t}\begin{pmatrix}1\\-2\end{pmatrix}$$

e) $Y(0) = c_1\begin{pmatrix}1\\1\end{pmatrix} + c_2\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}7\\-11\end{pmatrix}$

$$\begin{cases} c_1 + c_2 = 7 \\ c_1 - 2c_2 = -11 \end{cases} \Rightarrow 3c_2 = 18, \quad c_2 = 6,\ c_1 = 1.$$

Answer: $Y(t) = e^{5t}\begin{pmatrix}1\\1\end{pmatrix} + 6e^{2t}\begin{pmatrix}1\\-2\end{pmatrix}$.