Handout for HWK # 11 (due Monday, Oct. 22)

1. a) Verify that $y_1(t) = e^{-t}$ and $y_2(t) = e^{-4t}$ are solutions of $y'' + 5y' + 4y = 0$

b) Let C_1 and C_2 be two arbitrary constants. Verify by plugging in that $y(t) = C_1 e^{-t} + C_2 e^{-4t}$ is also a solution of $y'' + 5y' + 4y = 0$.

c) Explain why

$$Y(t) = \begin{pmatrix} y(t) \\ v(t) \end{pmatrix} = C_1 \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix} + C_2 \cdot \begin{pmatrix} e^{-4t} \\ -4e^{-4t} \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + C_2 e^{-4t} \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

must be a solution of the system

$$\begin{cases} \dfrac{dy}{dt} = v \\ dv/dt = -4y - 5v. \end{cases}$$

d) Find the solution of the system in c) that satisfies $y(0) = 3$, $v(0) = -2$.

2. a) Use the matrix notation to find the numbers λ_1 and λ_2 such that $\vec{Y}_1(t) = e^{\lambda_1 t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\vec{Y}_2(t) = e^{\lambda_2 t} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are solutions of the system $\dfrac{d\vec{Y}}{dt} = A\vec{Y}$ with $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$.

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b) Use the Linearity Principle to find the general solution of the system in a)

c) Find the particular solution of the system in a) that satisfies $Y(0) = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

d) Find the particular solution of the system in a) that satisfies $Y(0) = \begin{pmatrix} a \\ b \end{pmatrix}$ (Your answer will depend on constants a and b!)

3. Verify linearity of matrix multiplication:

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\vec{X} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $\vec{Y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

a) Verify that

$$A(\vec{X}+\vec{Y}) = A\vec{X} + A\vec{Y}$$ (Compute both sides and compare!)

b) Verify that

$$A(k\vec{X}) = kA\vec{X}$$, where k is an arbitrary constant.