Homework # 10 (Handout solutions) Total = 37

$\boxed{\#1}$ a) We plug in the solution into the system:

$$\begin{cases} (e^{4t}-2e^t)' = a(e^{4t}-2e^t)+2(e^{4t}+e^t) \\ (e^{4t}+e^t)' = e^{4t}-2e^t+d(e^{4t}+e^t) \end{cases}$$

$$\begin{cases} 4e^{4t}-2e^t = (a+2)e^{4t}+(-2a+2)e^t \\ 4e^{4t}+e^t = (d+1)e^{4t}+(d-2)e^t. \end{cases}$$

By comparing right and left sides of equations we conclude that $\boxed{a=2 \text{ and } d=3}$

5/5

b) $x(0) = e^{4\cdot 0}-2e^0 = -1$

$y(0) = e^{4\cdot 0}+e^0 = 2.$

2/2

$\boxed{\#\ 2}$ a) $\frac{dx}{dt} = 2x \Rightarrow x(t) = C_1 e^{2t}$

Then $dy/dt = x - 2y = C_1 e^{2t} - 2y.$

We will solve the linear equation

$$dy/dt + 2y = C_1 e^{2t}$$

by the method of integrating factors.

$$\mu(t) = e^{\int a(t)dt} = e^{\int 2dt} = e^{2t}$$

$$e^{2t}\, dy/dt + 2e^{2t}y = C_1 e^{2t}\cdot e^{2t}$$

$$(e^{2t}y)' = C_1 e^{4t} \Rightarrow e^{2t}y = \int C_1 e^{4t}dt = \frac{1}{4}C_1 e^{4t}+C_2$$

Thus, the general solution is $y = \frac{1}{4}C_1 e^{2t}+C_2 e^{-2t}.$

5/5 $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} C_1 e^{2t} \\ \frac{1}{4}C_1 e^{2t}+C_2 e^{-2t} \end{pmatrix}$

b) $\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} C_1 \\ \frac{1}{4} C_1 + C_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

3/3 Thus $C_1 = 1$ and $C_2 = -4 - \frac{1}{4} C_1 = -\frac{17}{4}$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} e^{2t} \\ \frac{1}{4} e^{2t} - \frac{17}{4} e^{-2t} \end{pmatrix}$$

$\boxed{\#3}$ a) Pluggin $y_1(t) = \cos(\beta t)$ into $y'' + 9y = 0$,

we get $(\cos(\beta t))'' + 9\cos(\beta t) = (-\beta^2 + 9)\cos(\beta t) = 0$

$\Rightarrow -\beta^2 + 9 = 0 \Rightarrow \beta = \pm 3$.

Thus $y_1(t) = \cos(3t)$.

5/5 In a similar way we obtain that $y_2(t) = \sin(3t$

b) $\begin{pmatrix} y_1(0) \\ y_1'(0) \end{pmatrix} = \begin{pmatrix} \cos(0) \\ -3\sin(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$,

2/2 $\begin{pmatrix} y_2(0) \\ y_2'(0) \end{pmatrix} = \begin{pmatrix} \sin(0) \\ 3\cos(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$.

c) $\begin{cases} dy/dt = v \\ dv/dt = -9y \end{cases}$.

3/3

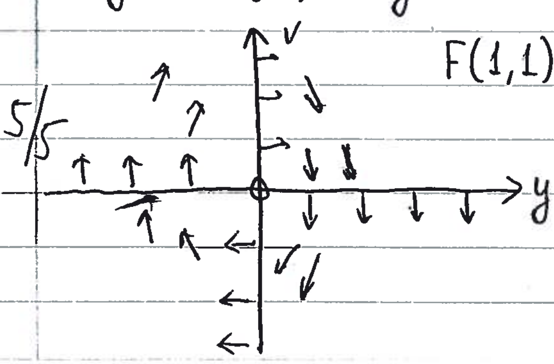
d) $Y_1(t) = \begin{pmatrix} y_1(t) \\ v_1(t) \end{pmatrix} = \begin{pmatrix} \cos(3t) \\ -3\sin(3t) \end{pmatrix}$

2/2 $Y_2(t) = \begin{pmatrix} y_2(t) \\ v_2(t) \end{pmatrix} = \begin{pmatrix} \sin(3t) \\ 3\cos(3t) \end{pmatrix}$

e) Vector field is

$F(y,v) = (v, -9y)$

5/5



$F(1,1) = (1,-9)$

f) $(y_1)^2 + \frac{(v_1)^2}{9} = [\cos(3t)]^2 + \frac{[-3\sin}{9}$

$= \cos^2(3t) + \sin^2(3t) = 1$.

5/5