Homework handout Hwk #10 (Due Wednesday, October 17.) ①

Instructions: You will turn in only problems from this handout. The rest of the assigned problems are for practice only, do not turn them in.

1. a) Find constants a and d so that

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} e^{4t} - 2e^{t} \\ e^{4t} + e^{t} \end{pmatrix}$$

is a solution of the system

$$\begin{cases} dx/dt = ax + 2y \\ dy/dt = x + dy. \end{cases}$$

See pp. 189-190

b) What is the initial condition for this solution?

2. a) Find the general solution of the following partially decoupled system:

$$\begin{cases} \dfrac{dx}{dt} = 2x, \\ dy/dt = x - 2y. \end{cases}$$

See pp. 192 - 194

b) Find the particular solution of this system satisfying $x(0) = 1$, $y(0) = -4$.

3. Consider differential equation representing simple harmonic oscillator

$$y'' + 9y = 0$$

See pp. 159 - 161

a) Find two different basic solutions of this equation. Seek solutions in the form $y_1(t) = \cos(\beta t)$ and $y_2(t) = \sin(\beta t)$.

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b) What are the initial conditions of these solutions?

c) Convert the differential equation $y'' + 9y = 0$ to a system in (y, v) variables, where $v = dy/dt$.

d) Use a) to find two different basic solutions of the system: $Y_1(t) = \begin{pmatrix} y_1(t) \\ v_1(t) \end{pmatrix}$ and $Y_2(t) = \begin{pmatrix} y_2(t) \\ v_2(t) \end{pmatrix}$.

e) Sketch the direction field for the system from c)

f) Verify that $(y_1(t))^2 + \frac{(v_1(t))^2}{9} = 1$

and use this to sketch the solution curve for $Y_1(t)$. (Hint: it will be an ellipse).