Section 3.3 (handout).

- Below are 3 examples of phase portraits of a system $\begin{cases} dx/dt = ax+by \\ dy/dt = cx+dy \end{cases}$, $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.
- We assume that A has two real eigenvalues $\lambda_1 \neq \lambda_2$
- If $|\lambda_1| < |\lambda_2|$, then λ_1 is the slow eigenvalue and λ_2 is the fast eigenvalue.

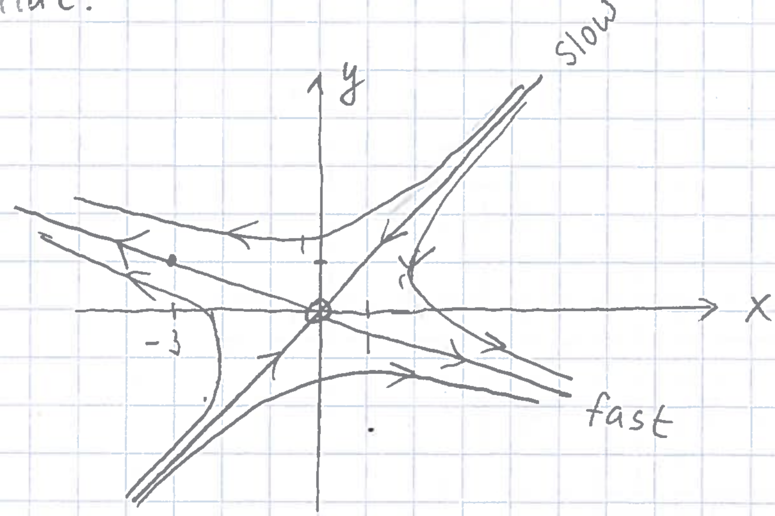
Example 1. (saddle)

$\boxed{\lambda_1 \cdot \lambda_2 < 0}$ Unstable.

$\lambda_1 = -2$, $\lambda_2 = 4$

$V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $V_2 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

slow fast

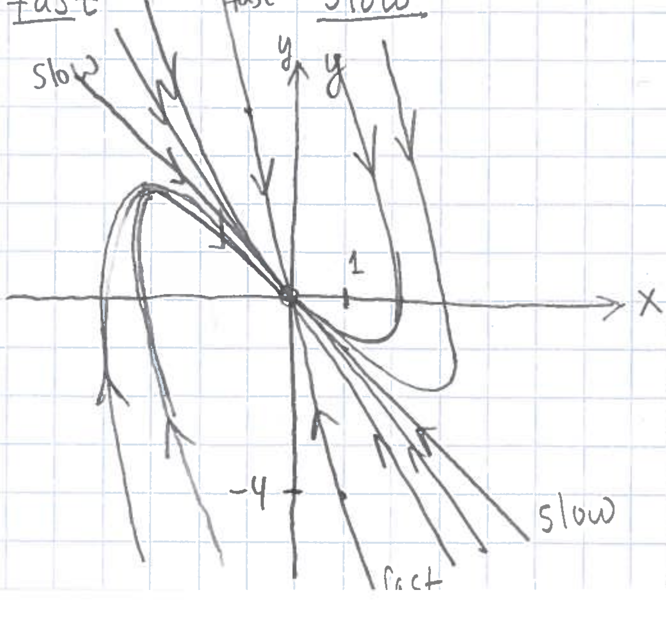


Example 2 (Sink) $\lambda_1 < 0, \lambda_2 < 0$

$\lambda_1 = -4$, $\lambda_2 = -1$ Stable.

$V_1 = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$, $V_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

fast slow



Example 3 (Source) $\lambda_1 > 0, \lambda_2 > 0$

$\lambda_1 = 2$, $\lambda_2 = 3$ Unstable

$V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $V_2 = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ fast

slow