Handout for Sec. 3.2 Finding the general solution of the system $\frac{dY}{dt} = AY$, where $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Step 1. Compute the characteristic polynomial:

$$p(\lambda) = \det \begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} = (a-\lambda)(d-\lambda) - bc$$

$$\boxed{p(\lambda) = \lambda^2 - (a+d)\lambda + ad - bc}$$

Step 2. Find the roots of the chapacteristic polynomial (use the quadratic formula, if necessary).

Let λ_1 and λ_2 be two roots, then λ_1 and λ_2 are eigenvalues of A

Step 3 For each eigenvalue λ_i, find corresponding eigenvector by solving the system

$$\boxed{\begin{cases} (a-\lambda_i)u + bv = 0 \\ cu + (d-\lambda_i)v = 0 \end{cases}}, \quad i = 1 \text{ or } 2.$$

This system will have a whole line of solutions, pick any nonzero solution as your eigenvector.

Step 4. If V_1 and V_2 are the eigenvectors you found in step 3 and $\lambda_1 \neq \lambda_2$, then V_1 and V_2 be linearly independent

The general solution will be

$$\boxed{Y(t) = k_1 e^{\lambda_1 t} V_1 + k_2 e^{\lambda_2 t} V_2},$$

where k_1 and k_2 are arbitrary constants.