Homework # 14 (due Friday, March 6)

1. Determine if sequence converges or diverges. If the sequence converges, find its limit.

a) $a_n = \dfrac{1-3n}{1+2n}$;

b) $a_n = \dfrac{n+(-1)^n}{n}$;

c) $a_n = \dfrac{n^3-1}{n^2+n+1}$;

d) $a_n = \dfrac{\sin n}{n}$;

e) $a_n = \ln n - \ln(n+1)$;

f) $a_n = \left(1+\dfrac{1}{2n}\right)^n$;

g) $a_n = \dfrac{3\cdot 10^n - 9^n}{8^n + 4\cdot 10^n}$;

h) $a_n = \dfrac{n!}{(n+2)!}$;

i) $a_n = \dfrac{n!}{n^2(n-2)!}$.

2. Find a formula for the n^{th} term in the sequence

a) $1, -4, 9, -16, 25, \ldots$;

b) $\frac{1}{2\cdot 3}, \frac{2}{3\cdot 4}, \frac{3}{4\cdot 5}, \frac{4}{5\cdot 6}, \ldots$

c) $a_1 = 3, \quad a_n - a_{n-1} = 5$ for all n;

d) $a_1 = 4, \quad \frac{a_n}{a_{n-1}} = \frac{1}{3}$ for all n.

3. Determine whether the sequence is monotonic and whether it is bounded:

a) $a_n = n e^{-n}$;

b) $a_n = \frac{\cos(\pi n)}{n}$;

c) $a_n = \frac{(n+1)!}{2^n}$.

4. Let $a_1 = 0, \quad a_{n+1} = \sqrt{8 + 2a_n}$.

a) Show that the sequence $\{a_n\}$ is increasing.

b) Show that $a_n \le 4$ for all n.

c) Show that $\lim_{n\to\infty} a_n = 4$.