Review for Final Exam (PDE)

Please review all the material below. In addition, please review homework and examples from the notes.

1. Section 1.1. PDE Models. Heat, wave, and Laplace equations. Initial conditions and boundary conditions. Verifying that a given function is a solution to an initial boundary value problem. Linear and nonlinear, homogeneous and nonhomogeneous equations. Superposition principle for linear equations.
2. Section 1.2. Conservation laws. Advection equation and its general solution. Advection – decay equation. Method of characteristics.
3. Section 1.3. Diffusion. Derivation of diffusion equation from a conservation law. Initial condition and types of boundary conditions for diffusion equation. Advection – diffusion and advection – diffusion – decay equation. Steady – state solutions.
4. Section 1.4. Diffusion and Randomness. Fundamental (point-source solution).
5. Section 1.5. Vibrations and Acoustics. Wave equation (without derivation). Initial conditions for wave equation. Types of boundary conditions for wave equation.
6. Section 1.7. Heat Conduction in Higher Dimensions. Divergence theorem. Derivation of diffusion equation in 3 dimensions. Initial condition and types of boundary conditions for diffusion equation (i.e. Dirichlet and Neumann). Laplace and Poisson equations.
7. Section 1.8. Laplace equation. The maximum principle. Laplacian in spherical and polar coordinates.
8. Section 1.9. Classification of PDE's as elliptic, hyperbolic, and parabolic. Changing variables in PDE's. Converting second order PDE's with constant coefficients into a canonical form.
9. Section 2.1. Heat kernel solution of the Cauchy problem for the heat equation.
10. Section 2.2. Derivation of D'Alembert's formula for solution of the Cauchy problem for the wave equation. Characteristics. Region of influence and domain of dependence.
11. Section 2.3. Definition of a well-posed problem. Examples of well-posed and ill-posed problems.

12. Section 2.4. Solving heat and wave equations on a half-line by the method of even/odd extensions.
13. Section 2.5. Solving heat and wave equation with sources. Finding particular solutions.
14. Section 2.7. Fourier transform, properties of Fourier transform. Solving PDE's by using Fourier transform.
15. Section 3.2. Fourier series. L^2 spaces. Complete orthogonal systems. Pointwise, mean-square, and uniform errors. Generalized Fourier series, Fourier coefficients. Best approximation theorem (statement of Theorem 3.8). Bessel's inequality. Parseval's equality.
16. Section 3.3. Classical Fourier Series. Fourier coefficients. Parseval's equality. Pointwise convergence theorem (Theorem 3.14) and Uniform convergence theorem (Theorem 3.15).
17. Section 4.1. Separation of variables. Eigenvalues and eigenfunctions. General solution.
18. Section 4.4. Solving Laplace equation in a disk by the method of separation of variables.
19. Section 4.5. Cooling of a sphere.
20. Section 4.7. Sources in Bounded domains (see notes).

Good luck!