**Homework 8 due March 21**

1. You need to know: all definitions and propositions (with proof) from section S.

2. Work out $\bigcap_{n=1}^{\infty} A_n$ and $\bigcup_{n=1}^{\infty} A_n$, where $A_n$ is defined as follows for $n \in \mathbb{N}$:

   (a) $A_n = \{x \in \mathbb{R} \mid x > n\}$

   (b) $A_n = \{x \in \mathbb{R} \mid \frac{1}{n} < x < \sqrt{2} + \frac{1}{n}\}$

   (c) $A_n = \{x \in \mathbb{R} \mid -n < x < \frac{1}{n}\}$

   (d) $A_n = \{x \in \mathbb{R} \mid \sqrt{2} - \frac{1}{n} < x < \sqrt{2} + \frac{1}{n}\}$.

3. Given a function $f$ and a subset $A$ of its domain, let $f(A)$ represent the range of $f$ over the set $A$; that is, $f(A) = \{f(x) \mid x \in A\}$.

   (a) Let $f(x) = x^2$. If $A = [0;2]$ and $B = [1;4]$, find $f(A)$ and $f(B)$. Does $f(A \bigcap B) = f(A) \bigcap f(B)$ in this case? Does $f(A \bigcup B) = f(A) \bigcup f(B)$?

   (b) Find two sets $A$ and $B$ for which $f(A \bigcap B) \neq f(A) \bigcap f(B)$.

   (c) Show that, for an arbitrary function $g : \mathbb{R} \to \mathbb{R}$, it is always true that $g(A \bigcap B) \subseteq g(A) \bigcap g(B)$ for all sets $A,\ B \subseteq R$.

   (d) Form and prove a conjecture about the relationship between $g(A \bigcup B)$ and $g(A) \bigcup g(B)$ for an arbitrary function $g$.

4. Give an example of each or state that the request is impossible:

   (a) $f : \mathbb{N} \to \mathbb{N}$ that is 1 to1 but not onto.

   (b) $f : \mathbb{N} \to \mathbb{N}$ that is onto but not 1 to 1.

   (c) $f : \mathbb{N} \to \mathbb{Z}$ that is 1 to 1 and onto.

5. Prove Lemma 6.1 on page 17 of Dr. Richardson's notes.