Homework due March 9

1. You need to know: Propositions R10, R11, R12 (all with proof) and additional definitions and propositions (to be announced on Tuesday).

2. Decide which of the following represent true statements about the nature of sets. For any that are false, provide a specific example where the statement in question does not hold.

   (a) If $A_1 \supseteq A_2 \supseteq A_3 \supseteq A_4 \supseteq ...$ are all sets containing an infnite number of elements, then the intersection $\bigcap_{n=1}^{\infty} A_n$ is infnite as well.

   (b) If $A_1 \supseteq A_2 \supseteq A_3 \supseteq A_4 \supseteq ...$ are all finite, nonempty sets of real numbers, then the intersection $\bigcap_{n=1}^{\infty} A_n$ is finite and nonempty.

   (c) $A \bigcap (B \bigcup C) = (A \bigcap B) \bigcup C$.

   (d) $A \bigcap (B \bigcap C) = (A \bigcap B) \bigcap C$.

   (e) $A \bigcap (B \bigcup C) = (A \bigcap B) \bigcup (A \bigcap B)$.

3. Produce an infinite collection of sets $A_1,\ A_2,\ A_3 ...$ with the property that every A_i has an infinite number of elements, $A_i \bigcap A_j = \oslash$ for all $i \neq j$, and $\bigcup_{n=1}^{\infty} A_n = \{1,\ 2,\ 3,\ ...\}$.

4. (De Morgan's Laws). Let A and B be subsets of real numbers.

   (a) If $x \in (A \bigcap B)^c$, explain why $x \in A^c \bigcup B^c$. This shows that $(A \bigcap B)^c \subseteq A^c \bigcup B^c$.

   (b) Prove the reverse inclusion $(A \bigcap B)^c \supseteq A^c \bigcup B^c$ and conclude that $(A \bigcap B)^c = A^c \bigcup B^c$.

5. (a) Show how induction can be used to conclude that $(A_1 \bigcup A_2 \bigcup ... \bigcup A_n)^c = A_1^c \bigcap A_2^c \bigcap ... \bigcap A_n^c$ for any finite positive integer n.

   (b) It is tempting to appeal to induction to conclude

$$\left(\bigcup_{i=1}^{\infty} A_i \right)^c = \bigcap_{i=1}^{\infty} A_i^c,$$

but induction does not apply here. Induction is used to prove that a particular statement holds for every value of n, but this does not imply the validity of the infinite case. To illustrate this point, find an example of a collection of sets $B_1,\ B_2,\ B_3, ...$ where

$$\bigcap_{i=1}^{n} B_i \neq \emptyset$$

is true for every n, but $\bigcap_{i=1}^{\infty} B_i = \emptyset$.

(c) Nevertheless, the infinite version of De Morgan's Law stated in (b) is a valid statement. Provide a proof that does not use induction.