Homework due March 2

1. You need to know: Propositions R1, R2, R3, R4, R5, R6 (all with proof), Axiom R7, Propositions R8, R10, R11, R12 (all with proof).

2. Prove that there are no rational numbers r and s such that
$$\sqrt{3} = r + s\sqrt{2}$$
.

3. (a) Prove that $\sqrt{5}$ is irrational.

   (b) Prove that $\sqrt{2} + \sqrt{3} + \sqrt{5}$ is irrational.

4. For each of the following statements, either prove it is true or give a counterexample to show it is false.

   (a) The product of two rational numbers is always rational.

   (b) The product of two irrational numbers is always irrational.

   (c) The product of two irrational numbers is always rational.

   (d) The product of a non-zero rational and an irrational is always irrational.

5. Express the decimal 1.813183... as a fraction $\frac{m}{n}$ (where m and n are integers).

6. Consider the equation $ax^2 + bx + c = 0$, where x is assumed to be a complex number and a, b, and c are fixed integers.

   (a) Prove or disprove that a solution to $ax^2 + bx + c = 0$ is always a rational number.

   (b) Prove or disprove that if one solution to $ax^2 + bx + c = 0$ is an integer, then all solutions of the equation are integers.

   (c) Prove or disprove that if one solution to $ax^2 + bx + c = 0$ is a rational number, then all solutions of the equation are rational numbers.