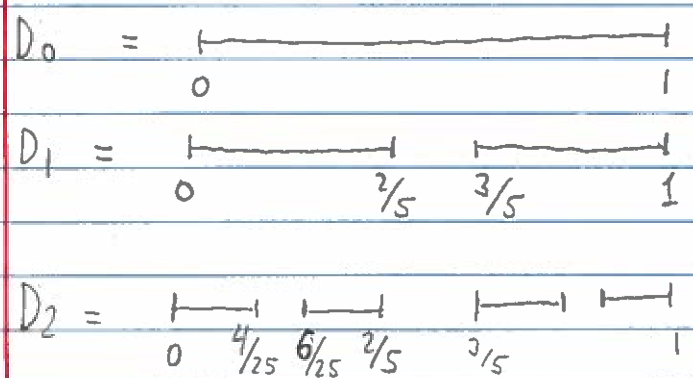
# Homework due Thursday, April 20

1. There will be a quiz on Thursday, April 20 on Cantor set and on arithmetic in different bases.

$\boxed{2.}$ Let $D$ be a "middle $1/5$ Cantor set", i.e. $D = \bigcap_{n=0}^{\infty} D_n$, where

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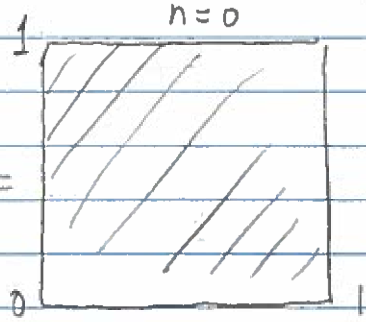
To get from $D_{n-1}$ to $D_n$ we remove middle $1/5$ of each of the intervals in $D_{n-1}$.

- a) Find a formula for the number of closed intervals in $D_n$ and for length of each interval. What is the total length of $D_n$?
- b) Does $D$ contain any intervals? Please explain
- c) How can one decide if $x$ in $D$? Please explain.
- d) Given two examples of points in $D$ which are not endpoints of any intervals in $D_n$.

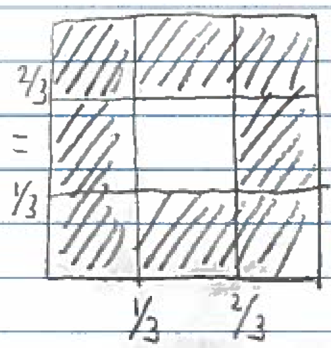
- e) Prove that $D$ is uncountable.
- f) What is the similarity dimension of $D$?

$\boxed{3.}$ Let $S$ be the Serpinski carpet,

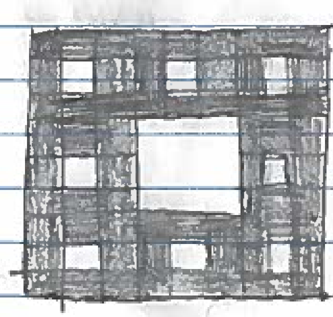
i.e. $S = \bigcap_{n=0}^{\infty} S_n$, where

$S_0 =$ 

$=$ solid square with side 1.

$S_1 =$ 

$=$ $S_0$ with midde square with side $1/3$ removed

$S_2 =$ 
$=$ $S_1$ with middle $1/9$ square removed from each of the squares in $S_1$

- a) Find a formula for the number of closed squares in $S_n$ and for the area of each square. What is the total area of $S_n$?
- b) Does $S$ contain any squares? Please explain.

c) How can one decide if a point $(x,y)$ is in $S$? Please explain.

d) Give two examples of points in $S$ which are not on the boundary of any of the removed squares.

e) Prove that $S$ and $[0,1]$ have the same cardinality.

f) What is the similarity dimension of $S$.

$\boxed{4.}$ A real number $r$ is called <u>algebraic</u> if $r$ is a solution of a polinomial equation $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$. For example $\sqrt{2}$ is algebraic since it solves $x^2 - 2 = 0$. Number $\sqrt{2}+1$ is also algebraic since it solves $x^2 - 2x - 1 = 0$, i.e. $(\sqrt{2}+1)^2 - 2(\sqrt{2}+1) - 1 = 0$. It is known that numbers $\pi$ and $e$ are not algebraic.

a) Prove that the number $\sqrt{2}+\sqrt{3}$ is algebraic.

b) Is the set of all algebraic numbers countable or uncountable?