Fourier Analysis Final Exam (review).

We will have a test on October 4. Please find below the list of topics you need to know for the test and the list of skill. Review lecture notes, and homework assignments.

1 Definition of an inner product space (real and complex). Examples of inner products. **Skill:** verifying properties of inner product.
2 Definition of L^2 and l^2 spaces. Verifying properties of inner products on these spaces.
3 Definitions of pointwise, uniform, and L^2 convergence. Theorem 0.10. Examples of each convergence. **Skill:** deciding if a given sequence converges pointwise, uniformly, or in L^2.
4 Schwartz and triangle inequalities: Theorem 0.11 (with proof).
5 Orthogonal vectors and orthogonal subspaces. Theorems 0.18 (with proof) and Theorem 0.25.
6 Orthogonal projection, orthogonal complement. Theorem 0.21. **Skill:** finding orthogonal projections to a given subspace, finding orthogonal complements.
7 **Skill:** Gram-Schmidt orthogonalization.
8 Fourier series on intervals of any length. Theorem 1.1 (with proof), Theorem 1.2 (with proof), and Theorem 1.4 (with proof). **Skill:** finding Fourier series of functions on an interval of any length.
9 Cosine and Sine expansions. **Skill:** finding cosine and sine expansions of even/ odd extensions of functions.
10 Pointwise convergence of Fourier series. **Skill:** graphing pointwise limit of Fourier series.
11 **Skill:** solving simple Partial Differential Equations by separating variables.
12 The complex form of Fourier series. Theorem 1.17, Theorem 1.18. Relations between coefficients of Fourier series in real and complex form. **Skill:** computing Fourier series in complex form.
13 Statement of Riemann – Lebesgue lemma (theorem 1.21, without proof). Convergence at point of continuity: Theorem 1.22. Convergence at a point of discontinuity: Theorem 1.28. Uniform convergence: Theorem 1.30. **Skill:** deciding when a given series convergent pointwise or uniformly to a given function, graphing pointwise/uniform limit of Fourier series.
14 Convergence in the mean. Definition of complete orthonormal set. Formula for Fourier coefficient (with proof). Bessel inequality (with proof). Parseval's equality (with proof). Lemma about 3 equivalent conditions of completeness (statement without proof). Theorem about completeness of standard trigonometric orthonormal system:
sin (nx), cos (nx) (statement). **Skill:** applying Parseval's theorem to obtain identities (see example 1.41).
15 Definition of Fourier transform and inverse Fourier transfroms. Inversion formula. **Skill:** finding Fourier transform of a given function.
16 Properties of Fourier transform (Theorem 2.6 with proof).
Skill: Applying properties of Fourier transform.
17 Fourier transform of convolution. Adjoint of Fourier transform (theorem 2.11). Plancherel Formula (theorem 2.12 with proof).
Skill: computing convolution of two functions.
18 Definition of a linear time-invariant transformation (filter). Lemma 2.16 and theorem 2.17 (both without proof) **Skill:** checking that a given transformation is linear and time-invariant. Finding a system function for a given filter.
19 Definition of a causal filter. Definition of a delta-sequence (class notes). Fourier transform of delta-function. Theorem 2.19 (with proof).
20 Definition of a frequency band-limited function. Nyquist frequency and Nyquist rate. Theorem 2.23 (with proof).

Good luck!