Lecture: 04/16 Calculus lecture.

Homework questions.

59 p. 634 $\sum_{n=1}^{\infty} \frac{10n+3}{n2^n}$

- Limit comparison with $\sum_{n=1}^{\infty} \frac{10\cancel{n}}{\cancel{n}\cdot 2^n} = \sum_{n=1}^{\infty} \frac{10}{2^n}$

- Ratio test $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1/2.$

- Direct comparison?

$$\sum_{n=1}^{\infty} \frac{10n+3}{n\cdot 2^n} < \sum_{n=1}^{\infty} \frac{10n+3n}{n\cdot 2^n}$$

$$= \sum_{n=1}^{\infty} \frac{13}{2^n} \leftarrow \text{geometric converges } r = 1/2.$$

64 $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

- Integral test

$f(x) = \frac{\ln x}{x^2} \rightarrow$ decreasing and positive. continutios

$$\int_1^\infty \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x^2}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$x = e^u$

$$= \lim_{t\to\infty} \int_0^{\ln t} \frac{u}{e^u}\,du$$

$$= \lim_{t\to\infty} \int_0^{\ln t} u e^{-u}\,du < \infty.$$

- Direct comparison test:

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^2} < \sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2} = \sum_{n=1}^{\infty}\left(\frac{1}{n^{3/2}} + \frac{4}{n^2}\right)$$

$\ln n < \sqrt{n}$, why this is true? $[4, +\infty)$

$f(x) = \ln x - \sqrt{x} < 0$

$f(1) = \ln 1 - \sqrt{1} = -1 < 0$

$f'(x) = \frac{1}{x} - \frac{1}{2\sqrt{x}}$, $x > 1$.

$= \frac{2\sqrt{x} - x}{2x\sqrt{x}} < 0$ $x > 4$.

$f(4) < 0$, $f'(x) < 0$ $x > 4$

$$\#61 \quad \sum_{n=1}^{\infty} \frac{\cos(n)}{3^n}$$

conv. absolutely.

conv.

Abs. conv. test: Direct comparison ✓

$$\text{conv.} \quad \sum_{n=1}^{\infty} \frac{|\cos(n)|}{3^n} < \sum_{n=1}^{\infty} \frac{1}{3^n}.$$

Will do 9.8, 9.9, 9.10, 9.7.

Sec. 9.8 (handout) Power series.

Main idea: Many functions can be represented by power series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

power series.

$$(e^x)' = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \dots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$= e^x$$

Definition: • A power series centered at $c=0$ is

$$a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n + \dots$$
$$= \sum_{n=0}^{\infty} a_n x^n$$

• A power series centered at c,

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \dots + a_n(x-c)^n + \dots$$

Here $a_1, a_2, \dots, a_n, \dots$ are numbers.

<u>Examples</u>: Find radius and interval of convergence for each power series below (For what values of x is the power series converge?)

a) $\displaystyle\sum_{n=0}^{\infty} \underbrace{\frac{x^n}{2^n \cdot n^2}}_{u_n(x)} = \sum_{n=0}^{\infty} \underbrace{\overset{a_n}{\frac{1}{2^n \cdot n^2}} \cdot x^n}_{u_n(x)} = \sum_{n=0}^{\infty} u_n(x)$

Power series centered at $c = 0$,

$$a_n = \frac{1}{2^n \cdot n^2}$$

Apply Ratio test to $\displaystyle\sum_{n=0}^{\infty} u_n(x)$:

$$\lim_{n\to\infty}\left|\frac{u_{n+1}(x)}{u_n(x)}\right| = \lim_{n\to\infty}\left|\frac{\frac{x^{n+1}}{2^{n+1}\cdot(n+1)^2}}{\frac{x^n}{2^n \cdot n^2}}\right|$$

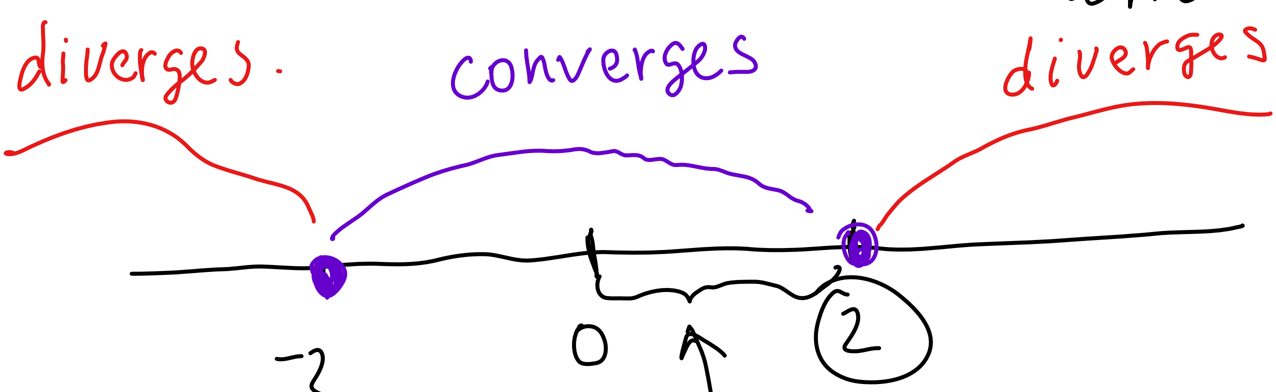
$$= \lim_{n\to\infty}\left|\frac{x^{n+1}}{x^n}\cdot\frac{2^n}{2^{n+1}}\cdot\frac{n^2}{(n+1)^2}\right|$$

$$= \lim_{n\to\infty} \left| x \cdot \frac{1}{2} \cdot \frac{n^2}{(n+1)^2} \right| = \frac{1}{2}|x| < 1$$

$$\boxed{|x| < 2}$$

So $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2^n \cdot n^2} = \begin{cases} \text{converges when } |x|<2 \\ \text{diverges when } |x|>2 \end{cases}$

don't know when $|x| = 2$



radius of convergence

$\boxed{R = 2}$

Interval of convergence is

$$-2 \le x \le 2$$

Need to investigate endpoints.

$x = 2$, then $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2 \cdot 2^n} = \sum_{n=1}^{\infty} \frac{\cancel{2^n}}{n^2 \cdot \cancel{2^n}}$

$$= \sum \frac{1}{n^2}$$

Converges
as p-serise $p = 2$.

converg. yes.

$x = -2$, then $\sum \frac{(-2)^n}{n^2 \cdot \cancel{2^n}} = \sum \frac{(-1)^n}{n^2}$, $(-2)^n = (-1)^n \cdot \cancel{2^n}$

converges by abs. conv. test

Interval of convergence is $[-2, 2]$.

Steps in finding interval of convergence

1. Apply ratio test to $u_n(x)$.
2. Find radius of convergence.
3. Find interval of convergence.

4. Investigate the endpoints of interval of convergence

$\underline{\text{Ex:}}$ $$\sum_{n=0}^{\infty} \underbrace{\frac{3^n (x-4)^n}{n}}_{u_n(x)}$$

1) Apply ratio test:

$$\lim_{n\to\infty} \left| \frac{u_{n+1}(x)}{u_n(x)} \right| = \lim_{n\to\infty} \left| \frac{\frac{3^{n+1}(x-4)^{n+1}}{n+1}}{\frac{3^n (x-4)^n}{n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{3^{n+1}(x-4)^{n+1}}{n+1} \cdot \frac{n}{3^n (x-4)^n} \right|$$

$$= \lim_{n\to\infty} \left| (x-4) \cdot 3 \cdot \frac{n}{n+1} \right| = 3|x-4| < 1$$

$$|x-4| < 1/3$$

$R = 1/3$ is the radius of conv