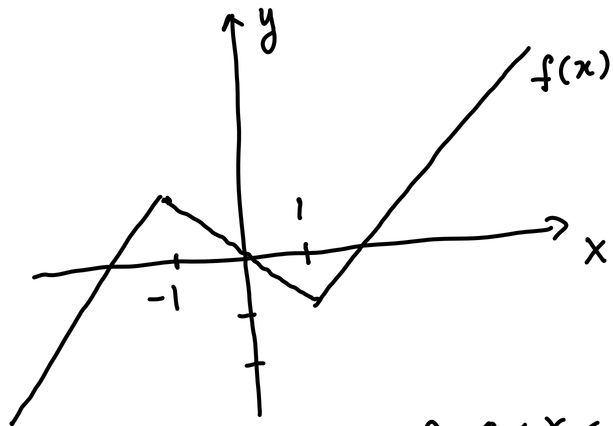
Lecture: Proof lecture on 04/15.

Comments on homework #18.

1 (ii) in the textbook.

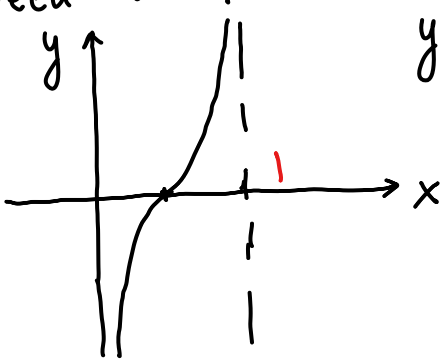
$$f(x) = \begin{cases} x-2, & \text{if } x>1 \\ -x, & \text{if } -1 \le x \le 1 \\ x+2, & \text{if } x<-1 \end{cases}$$



#2. $f(x) = \begin{cases} 2x, & \text{if } 0 \le x \le 1 \\ 1, & \text{otherwise} \end{cases}$ $\quad g(x) = \begin{cases} x^2, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$

So, $g(f(x)) = \begin{cases} g(2x), & \text{if } 0 \le x \le 1 \\ g(1), & \text{otherwise} \end{cases} = \begin{cases} (2x)^2, & \text{if } 0 \le x \le \frac{1}{2} \\ 0, & \text{if } \frac{1}{2} < x \le 1 \\ 1, & \text{otherwise.} \end{cases}$

#2 b) Need a function like:

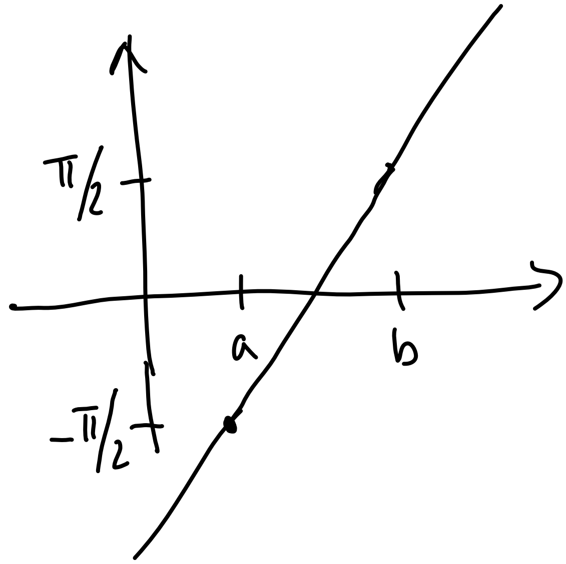


$y = \tan\left(\pi\left(x - \frac{1}{2}\right)\right) = b$

solve for b.

Questions about homework #19.

#2b



$y = mx + d = f(x)$

$f(a) = -\pi/2$

$f(b) = \pi/2.$

Proposition 19.2. Let S, T, U be 3 sets, $f: S \to T$ and $g: T \to U$ be 2 fcns.

Then

1) If f and g are both 1 to 1 then $g \circ f$ is also 1 to 1.
2) If f and g are both onto, then $g \circ f$ is onto.
3) If f, g are both bijections, then so is $g \circ f : S \to U$

Proof: 1) If $g \circ f(s_1) = g \circ f(s_2) \Rightarrow s_1 = s_2$ for all $s_1, s_2 \in S$.

Since $g: T \to U$ is one-to-one,

$$g(\underbrace{f(s_1)}_{t_1}) = g(\underbrace{f(s_2)}_{t_2}) \Rightarrow t_1 = t_2 \text{ or } f(s_1) = f(s_2)$$

Since $f: S \to T$ is one-to-one

$f(s_1) = f(s_2) \Rightarrow s_1 = s_2$. We are done.

2) Suppose that $u \in U$ is arbitrary, we want to find $s \in S$ so that

$$g(f(s)) = u.$$

Since g is onto, we can find $t \in T$

$g(t) = u$.

Since f is onto, we can find $s \in S$, s.t. $f(s) = t$.

Then $g(f(s)) = g(t) = u$. Done.

3) Obvious.

Chapter 21. Infinity.

How do we count?

- Set S has size n (it has exactly n elements) if there is a bijection from $\{1, 2, \dots, n\}$ to S.

- Notation: Size of set S is

$$|S| = \text{card}(S)$$

cardinality of set S.

Ex: $S = \{\Delta, \square, \star\}$, $|S| = \text{card}(S) = 3$.

$$\begin{matrix} \Delta & \square & \star \\ \updownarrow & \updownarrow & \updownarrow \\ 1 & 2 & 3 \end{matrix}$$

$\{1, 2, 3\}$.

- If S is a finite set, $|S| =$ number of elements in S.

- If $S = \emptyset$, $|S| = 0$

- $T = \{\emptyset\}$, $|T| = 1$.

- If S is not finite, it is called infinite, then $|S|$ can be represented by special symbols.

Ex: • $|\mathbb{N}| = \text{card}(\mathbb{N}) = \aleph_0$ (reads aleph zero)

• $|\mathbb{R}| = \text{card}(\mathbb{R}) = \aleph_1$

Def: Let A and B be two sets, we say that A and B have the same size (i.e. $|A| = |B|$ or $\text{card}(A) = \text{card}(B)$) if there is a bijection from A to B.

Examples: a)

$$\begin{array}{cccccc} A = \{ & 1, & x, & x^2, & x^3 & \} \\ & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ B = \{ & \triangle, & \square, & \star, & \bigcirc & \} \end{array}$$

$f: A \to B$, $f(1) = \triangle$, $f(x) = \square$, $f(x^2) = \star$, $f(x^3) = \bigcirc$.

$|A| = |B| = 4$.

b) $A = \{2, 4, 6, 8, \dots\}$

$B = \{1, 3, 5, 7, \dots\}$

Is there a bijection $f: A \to B$, yes:

$f(n) = n - 1$.