

Lecture: 04/13 Proof lecture
Comments about homework.

Prove that $f(A \cap B) \subseteq f(A) \cap f(B)$

Example exists that $f(A \cap B) \neq f(A) \cap f(B)$

$$f(x) = x^2, \quad A = \{1\}, \quad B = \{-1\}$$

$$\text{then } f(A \cap B) = \{\emptyset\}, \quad f(A) \cap f(B) = \{1\}$$

Notation issues: ~~$x \in \{f(x) \mid x \in A\}$~~
 ~~$x \in \underline{f(A)}$~~

$$y = f(x) : A \rightarrow B, \quad \underline{f(A) \subseteq B}$$

$$y \in f(A) \Leftrightarrow \exists x \in A \text{ s.t. } f(x) = y$$

$$\rightarrow y \in f(A \cap B) \Leftrightarrow \exists x \in A \cap B \text{ s.t.}$$

$$f(x) = y \Rightarrow x \in A \text{ and } x \in B \text{ s.t.}$$

$$f(x) = y \Rightarrow y \in f(A) \text{ and } y \in f(B)$$

$$x_1 \in A \Rightarrow f(x_1) = y$$
$$\text{and } x_2 \in B \Rightarrow f(x_2) = y$$

$$\Rightarrow y \in f(A) \cap f(B)$$

\Leftrightarrow

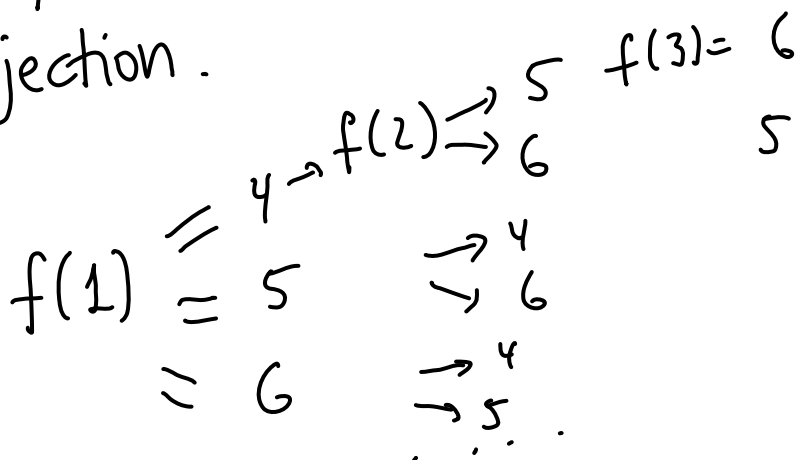
Functions (continued)

- $f: S \rightarrow T$ is one-to-one (injective) if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in S$.
(or $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$)
 - $f: S \rightarrow T$ is onto (surjective) if for any $t \in T \exists s \in S$ s.t. $f(s) = t$.
(or $f(S) = T$).
 - $f: S \rightarrow T$ is bijective if it both one-to-one and onto.
-

Ex: How many bijective functions are from $S = \{1, 2, 3\}$ to $T = \{4, 5, 6\}$

$f: S \rightarrow T$ How many different f 's ?

↑
bijection.



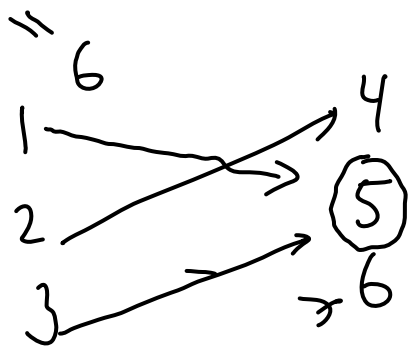
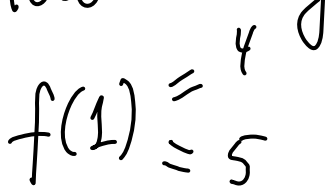
[3 choices for $f(1)$] · [2 choices for $f(2)$]
 [1 choice for $f(3)$] = 6.

How many injective $f: S \rightarrow T$ are?

Also 6.

How many surjective $f: S \rightarrow T$ are?

Also 6.



← not injective
 or surjective

Ex: $g: \{1, 2, 3\} \rightarrow \{4, 5\}$

a) How many surjective g ?

Exactly 6

$\{1, 2\} \rightarrow \{4, 5\}$ exactly 2.

$\left. \begin{array}{l} 1 \rightarrow 4 \quad 2 \rightarrow 5 \quad 3 \rightarrow 4 \\ 1 \rightarrow 5 \quad 2 \rightarrow 4 \quad 3 \rightarrow 5 \end{array} \right\} 4 \text{ cases.}$

$\{1, 3\} \rightarrow \{4, 5\}$ 4 cases

$\{2, 3\} \rightarrow \{4, 5\}$ 4 cases.

We double counting.

b) How many injective g ?

None.

c) How many bijective g ? None.

$h: \{1, 2, 3\} \rightarrow \{4, 5, 6, 7\}$

Inverse functions.

Definition: Let $f: S \rightarrow T$ be a bijection, then one can define an inverse function $g: T \rightarrow S$ as follows $g(t) = \textcircled{S}$ if and only if $f(s) = t$.

Solving $f(s) = t$ for s , this solution exists since f is onto and the solution is unique, since f is one-to-one.
" "
 $g(t) = s$.

Example: Let $f: [0, 2] \rightarrow [0, 4]$
 $f(x) = x^2$. Define inverse function $g: [0, 4] \rightarrow [0, 2]$.

$$g(3) = ? \quad f(x) = 3, \quad x^2 = 3,$$
$$x_1 = \sqrt{3}, \quad x_2 = \cancel{\sqrt{3}}, \quad \text{so } g(3) = \sqrt{3}.$$

You all know, $g(x) = \sqrt{x}$.
 $g(y)$ is a unique solution of
 $f(x) = y$.
" x^2 , $y = \sqrt{x}$

Def: Let S, T, U be sets,
 $f: S \rightarrow T$, $g: T \rightarrow U$. Then
their composition $h = g \circ f: S \rightarrow U$
defined by $h(s) = g(\underbrace{f(s)}_T)$ for
all $s \in S$.

Prop. 19.2 f, g are both one-to-one $\Rightarrow h$ is one-to-one
 f, g are both onto $\Rightarrow h$ also onto
 f, g are both bijections $\Rightarrow h$ is a bijection.

