Lecture: 04/06 Proof lecture

Hwk questions:

$A_1, A_2, A_3, \ldots, A_n,$

all infinite, disjoint

$$\bigcup_{n=1}^{\infty} A_n = \mathbb{N}.$$

1,3,5,7	2,6	4,12	8,24		. . .
0	1	2	3	4	

$$\boxed{n = 2^k \cdot m}$$

m is odd

n goes into A_{k+1}

Sec 17. More on sets.

Def. Given a subset A of $\mathbb{R}$ $(A \subseteq \mathbb{R})$, its complement A^c in $\mathbb{R}$ is $A^c = \{x \in \mathbb{R} \mid x \notin A\}$.

Ex: $(0,1)^c = (-\infty, 0] \cup [1, +\infty)$

$\{0\}^c = (-\infty, 0) \cup (0, +\infty)$

$\phi^c = \mathbb{R}$

$\mathbb{R}^c = \phi$

$A \subseteq \mathbb{R}$, $\boxed{(A^c)^c = A}$

Ex: $\boxed{(A \cup B)^c = A^c \cap B^c}$

Proof: $x \in (A \cup B)^c \iff x \notin A \cup B \iff$
$x \notin A$ and $x \notin B \iff x \in A^c$ and $x \in B^c$
$\iff x \in A^c \cap B^c$

$((A \cup B)^c)^c = (A^c \cap B^c)^c$

$\boxed{A \cup B = (A^c \cap B^c)^c}$ $(*) \leftarrow$ true.

$A^c = D$, $A = D^c$
$B^c = F$, $B = F^c$

$\rightarrow \boxed{D^c \cup F^c = (D \cap F)^c}$

Proposition 17.4. Let S be a set consisting of n elements. Then the total number of subsets of S is 2^n.

Ex: $S = \{1, 2, 3\}$, $n = 3$
must have 8 subsets.

$\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

Proof. Suppose S has n elements, label them $S = \{a_1, a_2, a_3, \ldots, a_n\}$
associate to each subset a string

of n zeroes an one's

<u>Ex:</u> $\{a_1, a_2, a_3\}$

$\emptyset \mapsto 000$

$\{a_1\} \mapsto 100$

$\{a_2, a_3\} \mapsto 011$

$\{a_1, a_2, a_3\} \mapsto 111$

$101 \mapsto \{a_1, a_3\}$.

Any subset $B \subset A = \{a_1, a_2, \ldots, a_n\}$ corresponds to a string $* * * \ldots *$

k^{th} place

$*$ is k^{th} place is 1 if $a_k \in B$
is 0 if $a_k \notin B$.

Now we need to count the number of strings of length n.

$$\underbrace{\underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \ldots \cdot \underline{2}}_{n \text{ place}} = 2^n$$

Chapter 19. Functions

Def: Let S and T be any sets. A function $f: S \to T$ is a rule that assigns to each $s \in S$ a single element $t \in T$, then one writes $f(s) = t$.

$$\boxed{f : S \to T}$$

Dirichlet
Weierstrass
1830 – 1890.

inputs are elt of S $\quad s \longrightarrow \boxed{\quad f \quad} \longrightarrow f(s) \quad$ outputs are elements of t

- Set S is called the domain of the function (set of all possible inputs)
- $f(S) = \{ f(s) \mid s \in S \}$ ← range or image of S

  ↑ set

  $f(S)$ = set of all possible outputs.

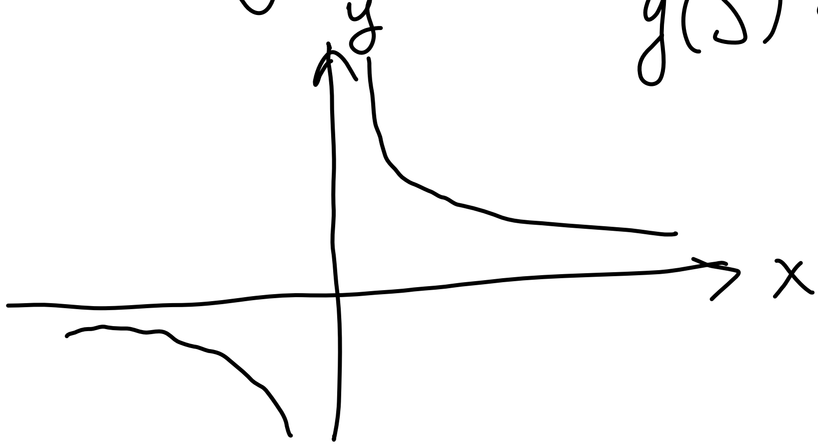
Notice that: $f(S) \subseteq T$

Ex: a) $g : S \to T = \mathbb{R}$, $g(x) = \frac{1}{x}$

Domain: $S = \{x \in \mathbb{R} \mid x \neq 0\}$

Range: $g(S) = \{x \in \mathbb{R} \mid x \neq 0\}$ $\qquad \frac{1}{x} = 0$

$g(S) \subset \mathbb{R}$. $\qquad$ no sol.



b) Dirichlet's function:

$d : \mathbb{R} \to \mathbb{N}$,

$$d(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

$d(1/3) = 1$, $d(\sqrt{2}) = 0$

Domain: $S = \mathbb{R}$, Range $f(S) = \{0, 1\} \subset \mathbb{N}$.

Graph of d:

Graph is not a good representation of this fcn.