Lecture on 03/27

Lecture:

Class announcements.

- Homework # 12 was graded and it is in your box folder.
- Hwk # 13 is due today at 3:30 p.m., are also due the corrections to hwk # 11 are also due the same time.
- Hwk # 14 due Monday will be posted later today.
- Test # 2 will be on Friday in class.

Comments about homework #12.

- Proving that $|x \cdot y| = |x| \cdot |y|$:

$$|x \cdot y| = |x| \cdot |y| \iff (|xy|)^2 = (|x| |y|)^2 \iff (xy)^2 = x^2 \cdot y^2$$

- When do we have equality in $|x+y| \leq |x| + |y|$?

equality when $x \geq 0$ and $y \geq 0$ or $x \leq 0$ and $y \leq 0$

then $2xy = 2|xy|$

- Solving $|x| - 2|x+3| = 2x$:

$$\underset{-3}{+} \quad \underset{0}{+}$$

3 cases:

a) $x \leq -3$

$-x - 2(-(x+3)) = 2x$

b) $-3 < x \leq 0$

$-x - 2(x+3) = 2x$

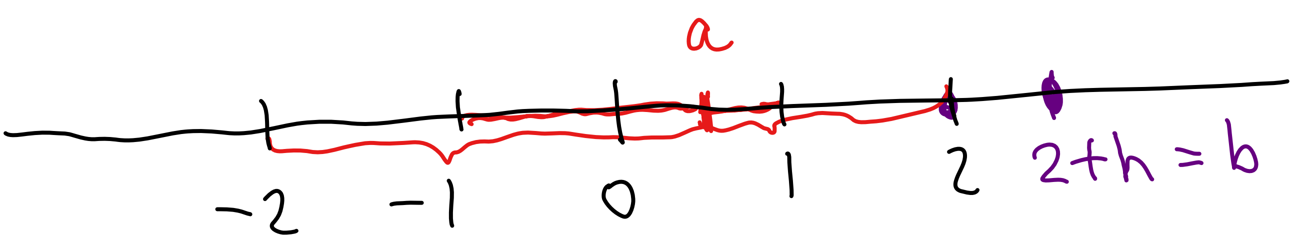
c) $x \geq 0$

$x - 2(x+3) = 2x$

Questions about hwk # 13.

\# 6

$$|x-2|+|x-1|+|x|+|x+1|+|x+2|=6$$



$\boxed{x=0}$ ← works!

$x=1$ $\quad |1-2|+|1-1|+1+|1+1|+|1+2|$

$= 1+0+1+2+3=7$

$$|a-1|+|a+1|=2$$

$$|a-2|+|a+2|=4$$

$$|a-2|+|a-1|+|a|+|a+1|+|a+2|=4+2+|a|$$

$$|b-2|+|b-1|+|b|+|b+1|+|b+2|$$

$$=4+3+2+1+0+5h>6$$

Chapter 17. More on sets.

Definition: A <u>set</u> is a well defined collection of distinct objects, called <u>elements of the set</u>.

<u>Set notation:</u> • Names of sets are usually capital letters: $A, B, \mathbb{R}, \mathbb{C}$

• Elements of set B are usually denoted as b:

$B = \{b_1, b_2, b_3\}$.

$b_2 \in B$ reads as "b_2 belongs to set B" or "b_2 is in B".

$\emptyset$ = empty set

<u>Example (sets of numbers):</u>

$\mathbb{N}$ = set of natural number $= \{1, 2, 3, 4, \dots\}$

$\mathbb{Z}$ = set of all integers $= \{0, 1, -1, 2, -2, \dots\}$

$\mathbb{Q}$ = set of all rational numbers $= \{\underbrace{m/n}_{\text{formula}} \mid \underbrace{m \in \mathbb{Z}, n \in \mathbb{N}}_{\text{plug into formula.}}\}$

$\mathbb{R}$ = set of all real numbers

$\mathbb{C}$ = set of all complex numbers $= \{a+ib \mid a, b \in \mathbb{R}\}$.

Example: $E = \{n \mid n = 2k, k \in \mathbb{Z}\}$

= set of all even numbers

$P_m = \{ p(x) \mid p(x)$ is a polynomial of degree $\leq m$ with real coefficients$\}$

Elt of P_m is $x^m + 3x + 4/3$, $m \geq 1$

$\left| \underline{a_m x^m + \ldots + a_0,\ a_m, \ldots, a_0 \in \mathbb{R}} \right.$

Unions and intersections.

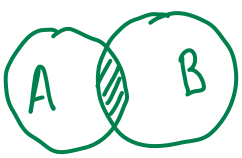
Definition: Let A and B be sets. The union of sets A and B written $A \cup B$ is the set consisting of all elements that lie in either A or B (or both):

$A \cup B$

Venn diagram

$$\boxed{A \cup B = \{ x \mid x \in A \text{ or } x \in B \}}$$

Definition: The intersection of A and B written $A \cap B$ is the set consisting of all elements that lie in both A and B.



$A \cap B$.

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}.$$

Ex: Consider

- $A = \{ x \in \mathbb{Q} \mid x > 0 \}$, • $B = \{ x \in \mathbb{Z} \mid x \geq 0 \}$
- $C = \{ n \mid 2k+1, k \in \mathbb{Z} \}$ • $D = \{ n \mid n = 3\ell, \ell \in \mathbb{Z} \}$.

What are.

$A \cup B = \{ x \in \mathbb{Q} \mid x > 0 \} \cup \{0\} = \{ x \in \mathbb{Q} \mid x \geq 0 \}$

- $A \cap B = \{ x \in \mathbb{Z} \mid x > 0 \}$

$C \cup D = \{ n \mid n \text{ is odd or } n = 3k, k \text{ is even} \}$.

$C \cap D = \{ n \mid n = 3 \cdot \ell,\ \ell \text{ is odd} \} =$

$A \cap B \cap C = \{n \mid n = 2k+1, \; k = 0, 1, \ldots\}$ $\left(= \{n \mid n = 3(2k+1), k \in \mathbb{Z}\}\right)$.

$A \cap E = 2\mathbb{N}$

$C \cup P_m = P_m$, $\quad C \subset P_m$ for all m.

$C \cap P_m = C$

Definition (Subsets): • set A is a subset of a set B written $A \subseteq B$, if $\forall x \in A$, $x \in A \Rightarrow x \in B$.

• set A is a proper subset of B written $A \subset B$, if $A \subseteq B$ and $A \neq B$.

• sets A and B are equal, written $A = B$, if $A \subseteq B$ and $B \subseteq A$.