Lecture:

Announcements: • Quiz today at 1.30 p.m.
Due 2 p.m (time and a half 2:15 pm)
• Solutions to hwk 15 and quiz 14 are available on homework page.
• Next hwk is due Monday and a quiz due Monday.

Last time:
• Geometric series:

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + \ldots + ar^n + \ldots$$

$$= \begin{cases} \frac{a}{1-r}, & |r| < 1 \\ \text{diverges}, & \text{otherwise} \end{cases}$$

• Telescoping series:

Ex: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1$

- Divergence test: $\lim_{n\to\infty} a_n = 0 \Rightarrow \sum_{n=1}^{\infty} a_n$ diverges

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Homework questions:

#97

$$1 = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) \neq \sum \frac{1}{n} - \sum \frac{1}{n+1}$$

↑ converges

$$\infty = \sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges} \qquad \sum_{n=1}^{\infty} \frac{1}{n+1} \text{ also} = \infty$$

diverges.

$$0 = \sum_{n=1}^{\infty} \underbrace{(1-1)}_{0} \neq \underbrace{\sum_{n=1}^{\infty} 1}_{\infty} - \underbrace{\sum_{n=1}^{\infty} 1}_{\infty}$$

#49 $\sum_{n=2}^{\infty} \frac{n}{\ln n}$ ← diverges

$$a_n = n/\ln n \qquad \lim_{n\to\infty} n/\ln n = \lim_{x\to\infty} x/\ln x \overset{\text{l'Hop}}{=}$$

$= +\infty \neq 0$, by div. test $\sum_{n=2}^{\infty} n/\ln n$ diverges.

Properties of infinite series.

If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B (i.e. $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$), then

1) $\sum_{n=1}^{\infty} c a_n = cA$;

2) $\sum_{n=1}^{\infty} (a_n \pm b_n) = A \pm B$.

Ex 1: $4 = \sum_{n=1}^{\infty} \left(2\left(\frac{1}{2}\right)^n + 4\cdot\left(\frac{1}{3}\right)^n\right) = 2\cdot\frac{1/2}{1-1/2} + 4\cdot\frac{1/3}{1-1/3} = 2+2 = 4$

$$2\underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n}_{\text{conv.}} + 4\underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n}_{\text{conv.}}$$

$$= 2\cdot\left(\frac{1}{2} + \frac{1}{2^2} + \dots\right) + 4\left(\frac{1}{3} + \frac{1}{3^2} + \dots\right) = 4$$

Ex 2: $\underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)}_{\text{converges to 1.}} \overset{?}{\cancel{=}} \underbrace{\sum_{n=1}^{\infty}\frac{1}{n}}_{\text{diverges}}^{=\infty} - \underbrace{\sum_{n=1}^{\infty}\frac{1}{n+1}}_{\text{diverges}}^{=\infty}$

Sec. 9.3 (handout) Sec 5.3 (textbook)

Integral test and p-series.

- Integral test can be applied to series like $\sum_{n=1}^{\infty} \frac{1}{n^2}$ or $\sum_{n=1}^{\infty} \frac{\ln n}{n}$,

where $\sum_{n=1}^{\infty} \frac{1}{n^2} = \sum_{n=1}^{\infty} f(n)$, $f(x) = \frac{1}{x^2}$

and $\sum_{n=1}^{\infty} \frac{\ln n}{n} = \sum_{n=1}^{\infty} g(n)$, $g(x) = \frac{\ln x}{x}$

Integral test: Suppose that

$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} f(n)$, i.e. $a_n = f(n)$

and the function $f(x)$ is both positive and decreasing for $x \geq 1$.

Then there are exactly two

possibilities:

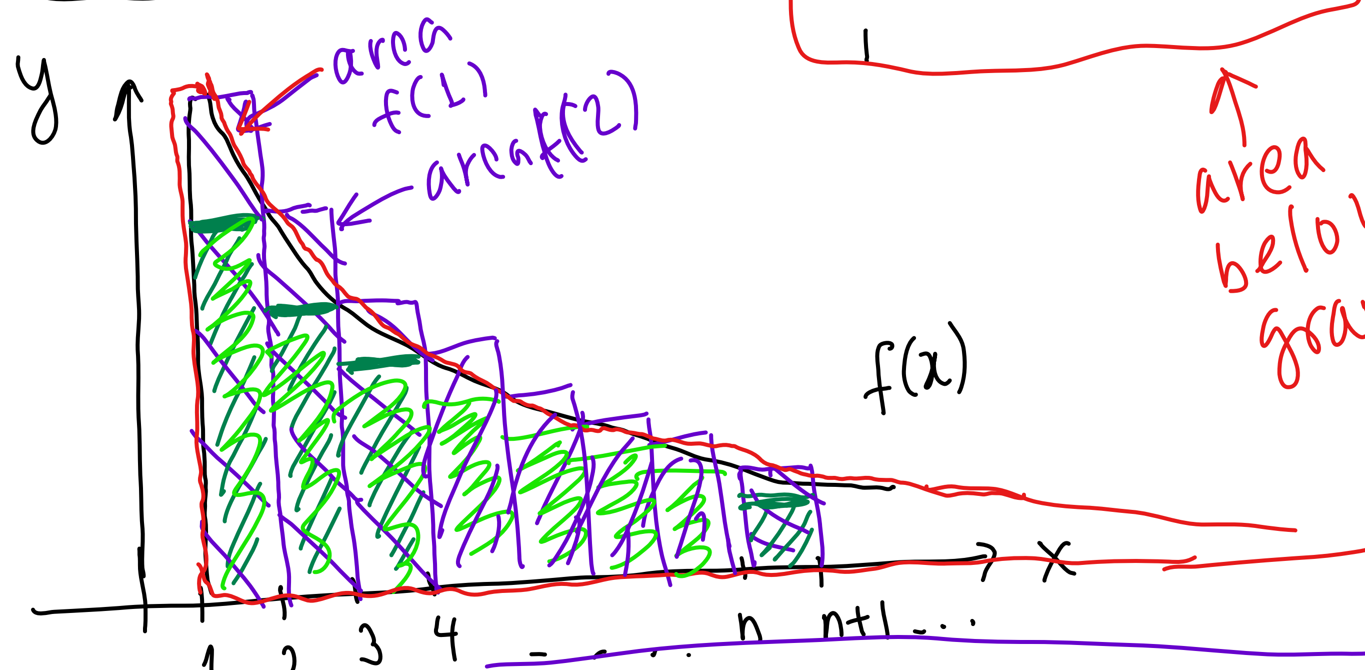
a) $\sum_{n=1}^{\infty} a_n$ converges and $\int_1^{\infty} f(x)\,dx = S < \infty$ both series and integral converge

$\left(\text{and } S < \sum_{n=1}^{\infty} a_n < S + a_1\right)$

b) $\sum_{n=1}^{\infty} a_n = \infty$ and $\int_1^{\infty} f(x)\,dx = \infty$,

both series and integral diverge.

Explanation:



$\boxed{\int_1^{\infty} f(x)\,dx = S}$

area below graph.

$f(2)\cdot 1 + f(3)\cdot 1 + \ldots + f(n)\cdot 1 + \ldots < \int_1^{\infty} f(x)\,dx < f(1)\cdot 1 + f(2)\cdot 1 + \ldots + f(n)\cdot 1 + \ldots$

So $$\int_1^\infty f(x)\,dx < a_1 + a_2 + \dots + a_n + \dots = \sum_{n=1}^\infty a_n$$

and $$\sum_{n=1}^\infty a_n < \int_1^\infty f(x)\,dx + a_1 .$$