HOMEWORK 20
DIFFERENTIAL EQUATIONS
DUE 11-21

Show your work!

(1) Consider the differential equation

$$u'' + 4u' + 5u = u_3(t), \quad u(0) = 1,\ u'(0) = -3.$$

(a) Find the formula for the solution $u(t)$ when $0 < t < 3$.

(b) Assuming that $u(t)$ is continuous at $t = 3$, find the formula for the solution $u(t)$ when $t > 3$. (Hint: Use continuity to find the initial condition.)

(2) Suppose that y_1 is any function, and $y(t) = e^{at}y_1(t)$ for some constant a. Use the integral definition of the Laplace transform to prove that $Y(s) = Y_1(s-a)$, where $Y_1 = \mathcal{L}\{y_1\}$ and $Y = \mathcal{L}\{y\}$. (In words, multiplication by an exponential in the t-domain corresponds to a shift in the s-domain.)

(3) Use integration by parts, and the integral definition of the Laplace transform, to show that

$$\mathcal{L}\{y'\} = s\mathcal{L}\{y\} - y(0).$$

What do you need to assume about $e^{-s(\infty)}y(\infty)$? (The technical details are in #6.1.29.)

- **Five** book problems: #6.1.16, 23; #6.2.24; #6.3.4, 6.