HOMEWORK 18
DIFFERENTIAL EQUATIONS
DUE 11-07

Show your work! In class, we claimed that just about everything one needs to know about the Laplace transform can be boiled down to two rules:

$$\mathcal{L}\left\{e^{at}\right\} = \frac{1}{s-a} \quad \text{and} \quad \text{if } \mathcal{L}\left\{f(t)\right\} = F(s), \text{ then } \mathcal{L}\left\{f'(t)\right\} = sF(s) - f(0).$$

For the handout problems, you must show your work; just giving the answer using the chart on p. 321 is *not* enough. You also must give the answer as a single fraction, simplified as far as possible.

(1) For this problem, you may use #3.3.29 (without re-solving it). Give your answers as single fraction, simplified as far as possible.
   (a) Compute $\mathcal{L}\left\{\sin(\omega t)\right\}$ and $\mathcal{L}\left\{\cos(\omega t)\right\}$.
   (b) Compute $\mathcal{L}\left\{e^{at}\sin(bt)\right\}$ and $\mathcal{L}\left\{e^{at}\cos(bt)\right\}$.

(2) Find a formula for $\mathcal{L}\left\{f''(t)\right\}$ in terms of $F(s)$, $f(0)$, and $f'(0)$. (Hint: Think of it as $\mathcal{L}\left\{g'(t)\right\}$, where $g(t) = f'(t)$.)

(3) (a) Show that, if $g'(t) = f(t)$, then $\mathcal{L}\left\{g(t)\right\} = \frac{1}{s}\big(F(s) + g(0)\big)$. (Hint: $F(s) = \mathcal{L}\left\{g'(t)\right\}$.)
   (b) Use part (a) to compute $\mathcal{L}\left\{t\right\}$.

(4) In class, we saw that, if

$$3y' + 4y = e^{2t}, \quad y(0) = 1,$$

then $Y = \dfrac{3s-5}{(s-2)(3s+4)}$.
   (a) What is a function whose Laplace transform is $\dfrac{1}{3s+4}$? (Hint: Factor out $\dfrac{1}{3}$.)
   (b) Use partial fractions to write

$$\frac{3s-5}{(s-2)(3s+4)} = \frac{A}{s-2} + \frac{B}{3s+4}.$$

   (c) Use your answer to (b) to find y.
   (d) Check that your answer to (c) is actually a solution to the original problem.

- **Two** book problems: #6.2.1, 3.