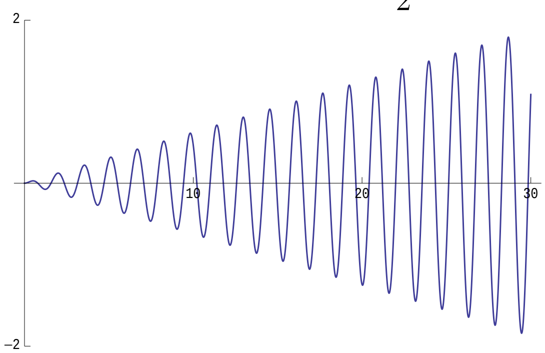
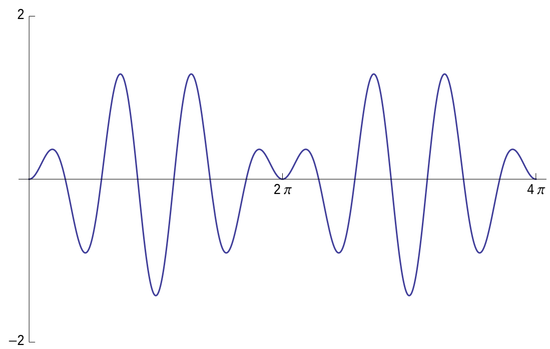
HOMEWORK 17
DIFFERENTIAL EQUATIONS
DUE 11-05

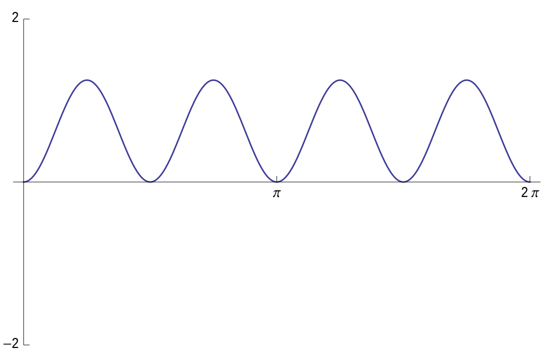
Show your work!

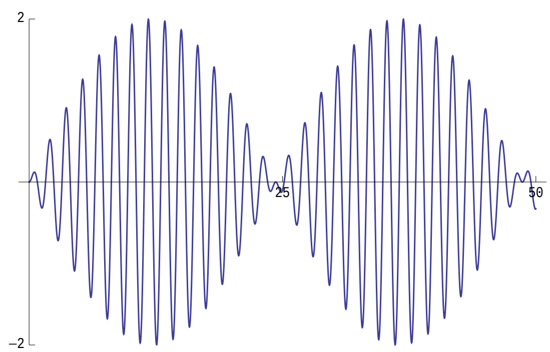
(1) Six second-order equations and four graphs of solutions are given below. For each graph, determine the differential equation of which it is a solution, and explain how you could figure out (without using a computer or calculator) that your answer was correct.

(i) $y'' + 16y = 10$ (ii) $y'' + 16y = -10$
(iii) $y'' + 16y = 5\cos(3t)$ (iv) $y'' + 14y = 2\cos(4t)$
(v) $y'' + 16y = \dfrac{1}{2}\cos(4t)$ (vi) $y'' + 2y' + 16y = \cos(4t)$

(a)


(b)


(c)


(d)


(2) Suppose that the suspension system of a typical car behaves like an undamped harmonic oscillator whose natural period is 2 sec. The developer of a certain neighbourhood wants to place speed bumps so that a car driving at 10 mph will bounce more and more with each bump. How far apart should the bumps be placed?

(3) Let ω_{nat} be the natural frequency of a certain undamped harmonic oscillator, and ω the frequency of a forcing function applied to it. Put $\alpha = \frac{1}{2}(\omega + \omega_{\text{nat}})$ and $\beta = \frac{1}{2}(\omega - \omega_{\text{nat}})$.
   (a) Show that $e^{i\omega t} - e^{i\omega_{\text{nat}}} = e^{i\alpha t}(e^{i\beta t} - e^{-i\beta t})$.
   (b) Compute the real part of $e^{i\omega t} - e^{i\omega_{\text{nat}} t}$.
   (c) Compute the real part of $e^{i\alpha t}(e^{i\beta t} - e^{-i\beta t})$.
   (d) Explain how your work above can be used to express $\cos(\omega t) - \cos(\omega_{\text{nat}} t)$ as a product of two sine waves. **Do not** just cite the identity from class.

- **Four** book problems: #3.8.5, 7, 18, 19.