HOMEWORK 14
DIFFERENTIAL EQUATIONS
DUE 2013-10-15

Show your work!

(1) Consider the example from class of a spring attached at one end to the ceiling, and at the other to a mass of weight 10 lb. The rest length of the spring–mass system is 2 in. Suppose that, when the mass has a velocity v, it experiences (in addition to the spring force) a damping force of magnitude γv directed opposite to the velocity.

   (a) Suppose that $\gamma = 1$ lb · sec/in. Find the general solution of the resulting differential equation for the downward displacement u from equilibrium. In this situation, we say that the spring is *overdamped*.

   (b) Suppose that the spring is initially compressed to the ceiling, and then thrown downward with a velocity of 1 ft/sec. Find the particular solution u (that is, find the constants in your answer to (a)).

   (c) Give an example of an initial condition that will cause the displacement of the spring to tend to 0 more rapidly than in (b). (HINT: Make a choice of constants that 'kills' one of the terms in (a).)

(2) Consider the same spring as in #1.

   (a) As γ gets larger, we switch from exponentially decaying oscillations (as when $\gamma = 1/2$ lb · sec/in) to pure exponential decay (as when $\gamma = 1$ lb · sec/in). What is the cut-off value for γ? At this level of damping, we say that the spring is *critically damped*.

   (b) Find the general formula for the displacement u of the critically damped spring from equilibrium.

- **Five** book problems: #3.7.5, 11, 13, 14, 19. The *period* T of an oscillation is the time required for one complete cycle; it is related to its (angular) frequency ω_0 by $T = 2\pi/\omega_0$. See (18), p. 197. The *quasi-frequency* μ of an exponentially decaying oscillation $e^{-at}(C_1 \cos(bt) + C_2 \sin(bt))$ is the coefficient $\mu = b$ occurring in the trigonometric terms, and the *quasi-period* is $T_\mathrm{d} = 2\pi/\mu$. See p. 199.