HOMEWORK 13
DIFFERENTIAL EQUATIONS
DUE 2013-10-10

Show your work!

(1) The differential equation $y'' + 5y' + 6y = 0$ has $y = e^{-2t}$ as one solution. Use reduction of order to find the general solution. (You **must** use reduction of order, not the method of §3.1.)

(2) This is problem #3.3.34, but broken down into more steps. It's another illustration of the power of a change of variables, slightly more involved than reduction of order.

(a) Suppose that you have three variables y, x, and t. The chain rule says that

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t}$$

Explain why the chain rule also says that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{\mathrm{d}^2y}{\mathrm{d}x^2} \cdot \frac{\mathrm{d}x}{\mathrm{d}t}.$$

(b) Suppose that $x = \ln(t)$. Use your answer to (a) and the product rule to show that

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \cdot \frac{1}{t} \quad \text{and} \quad \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = \frac{\mathrm{d}^2y}{\mathrm{d}x^2} \cdot \frac{1}{t^2} - \frac{\mathrm{d}y}{\mathrm{d}x} \cdot \frac{1}{t^2}.$$

(c) Use your answer to (b) to show that, if

$$t^2\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \alpha t\frac{\mathrm{d}y}{\mathrm{d}t} + \beta y = 0, \quad t > 0,$$

where α and β are constants, and $x = \ln(t)$, then

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (\alpha - 1)\frac{\mathrm{d}y}{\mathrm{d}x} + \beta y = 0$$

(which is a constant-coefficient, homogeneous, 2nd-order linear equation).

- **Four** book problems: #3.3.35; #3.4.14, 16, 27.