**HOMEWORK 8**
**DIFFERENTIAL EQUATIONS**
**DUE 2013-09-17**

**Show your work!**

(1) Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 0.1y\left(1 - \frac{y}{5}\right).$$

(a) Find the general solution of this equation in *implicit* form.

(b) Find the general solution in *explicit* form.

(c) Use your answer to (b) to show that, if $y(0) > 0$, then $\lim_{t\to\infty} y = 5$.

(d) (+1 **bonus**) What changes in your answer to (c) if $y(0) < 0$? (HINT: Try a specific choice of initial condition.)

(2) Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 0.1y\left(\frac{y}{3} - 1\right)\left(1 - \frac{y}{5}\right).$$

We call $y = 3$ the *threshold* population, and $y = 5$ the *carrying capacity*.

(a) Describe the long-term behaviour of populations with varying initial sizes. Why are the names 'threshold' and 'carrying capacity' appropriate?

(b) What is the formula for $\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$ in terms of $y$?

(c) What are the 'tipping points' for $y$?

- **Five** book problems: #2.6.1, 4, 7, 15, 19. For #2.6.1, 4, 7, the equation is exact; you do not need to test it. For #2.6.19, you do not need to test that the original equation is *not* exact.