HOMEWORK 6
DIFFERENTIAL EQUATIONS
DUE 2013-09-10

Show your work!

(1) We saw in class that, if a rocket is projected upwards from the Earth's surface with an initial velocity of v_{init}, and if we ignore thrust and air resistance, then the velocity v when the height is h satisfies

$$\frac{1}{2}v^2 = \frac{gR^2}{R+h} + \frac{1}{2}v_{\text{init}}^2 - gR,$$

where $g \approx 9.807$ m/s^2 is acceleration due to gravity at the Earth's surface, and $R \approx 6.378 \times 10^3$ m is the radius of the Earth.

(a) Suppose that $v_{\text{init}} = \sqrt{2gR}$. (This is called *escape velocity*.) Find a formula for the velocity v in terms of the height h.

(b) Use (a) to write a differential equation for h in terms of t. (This is easy!)

(c) Solve your equation in (b) to show that

$$h = R^{1/3}\Big[\Big(3\sqrt{\frac{gR}{2}}t + R\Big)^{2/3} - R^{2/3}\Big].$$

(2) Read #2.3.27.

(a) The mass of the sphere is the volume of the sphere times the density of the sphere. Find a formula for the mass of the sphere in terms of a and ρ.

(b) The mass of the fluid displaced is the volume of the sphere times the density of the fluid. Find a formula for the mass of the fluid displaced in terms of a and ρ'.

(c) Use your answers to (a) and (b) to write down a formula for the total force on the sphere. (HINT: The magnitude of buoyant force is the mass of the fluid displaced times g; the magnitude of the weight is the mass of the sphere times g; and the magnitude of the resistive force is given.)

(d) Use (c) to write a differential equation for the velocity v in terms of t.

(e) Solve #2.3.27 (both parts).

- **Two** book problems: #2.3.24, 25(a). (HINT: For #2.3.24, pay attention to units! For part (c), imitate handout problem #1 to find a formula for x.)